MTH 267, Quiz #1, Fall 2022 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Differentiate the function $y = t^2 \ln(\cos 2t)$.

$$y' = 2t \ln(\cos 2t) + t^2 \cdot \frac{2\sin 2t}{\cos 2t}$$

$$= 2t \ln(\cos 2t) + 2t^2 \tan 2t$$

2. Integrate $\int \frac{t}{1-t} dt$.

$$t \overline{)-t+1} \quad -1$$
$$\underline{+t}$$
$$1$$

$$\int -1 + \frac{1}{1-t}\, dt = -t - \ln|1-t| + C$$

3. Use technology to plot the direction field for the differential equation $\frac{dy}{dt} = y(y - 2t)$. Label and equilibria or nullclines. Include in your sketch sample trajectories of solutions in each region of the field.

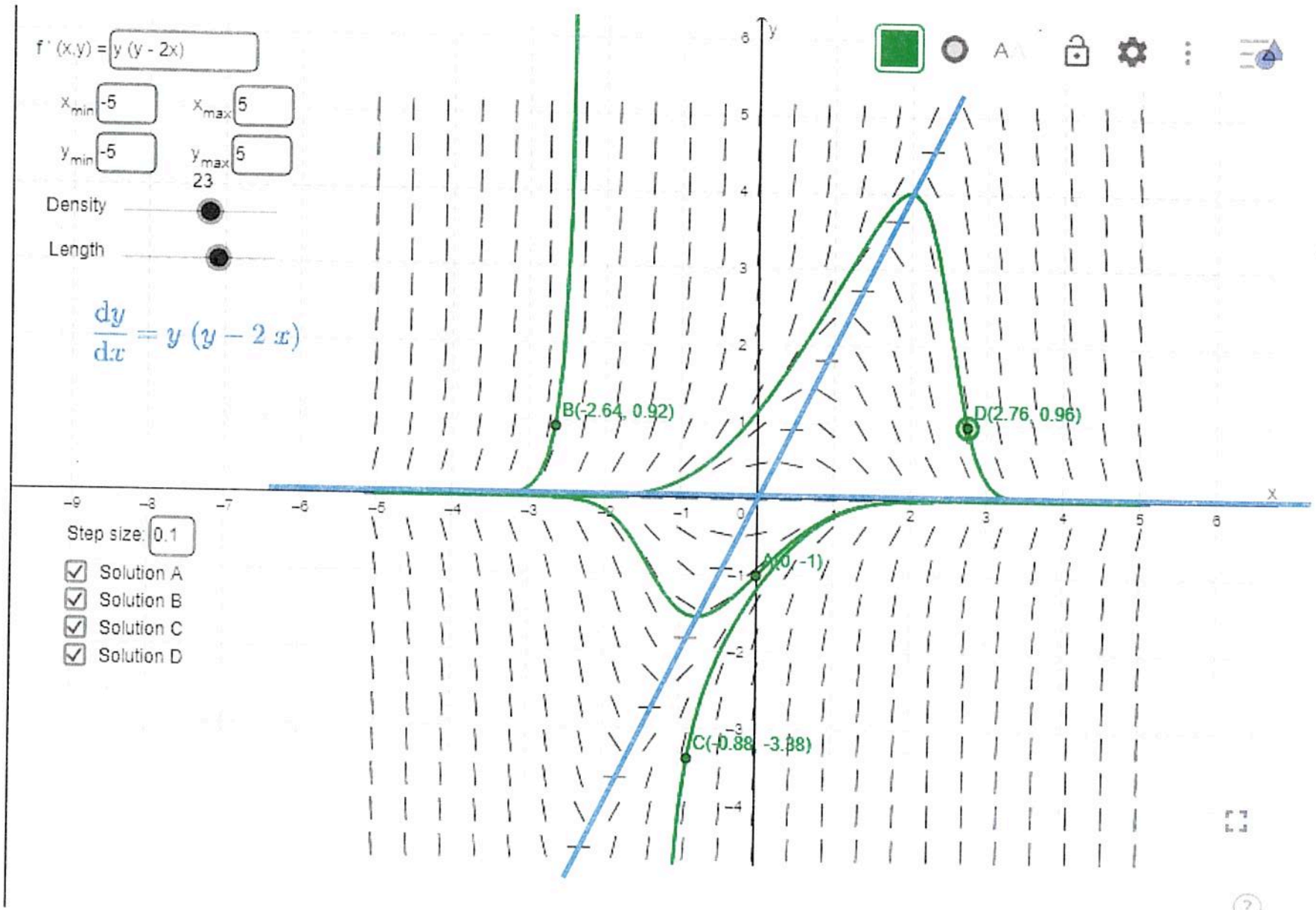
attached

equilibrium $y=0$

nullcline $y = 2t$

blue lines are nullclines/equilibria

green lines are sample trajectories


f ' (x,y) = y (y - 2x)
x min -5
x max 5
y min -5
y max 5
23
Density
Length
$\frac{dy}{dx} = y\,(y - 2\,x)$
Step size 0.1
Solution A
Solution B
Solution C
Solution D
B(-2.64, 0.92)
D(2.76, 0.96)
A(0, -1)
C(-0.88, -3.38)
x
y