MTH 267, Numerical Methods Lab #2, Fall 2022 Name _____

Instructions: This lab will be done primarily in Excel. Detailed instructions for the lab are outlined below. You will need to download the model file and complete the indicated problems but modifying the model to fit the new problem. To submit the lab, submit the Excel file you generated, and the completed questions below.

1. Open the model file entitled **267lab2.xlsx**. The problem modeled in the file on Sheet1 is applying Euler's method to $y' = \sqrt{t+y}$, with $y(0) = 2$ as the initial condition. The initial step size is $\Delta t = h = 0.1$. The second line of the file calculates the slope and the next y_{n+1} value using Euler's method. Line three copies the necessary values from line one, and then completes the next calculation. The first line contains the column labels. This lab follows the model from Lab #3 which was also in Excel, but now we will use multiple methods of estimation on the same set of problems. Sheet2 is set up to estimate using the Modified Euler's method, and Sheet3 is set up for Runge-Kutta.

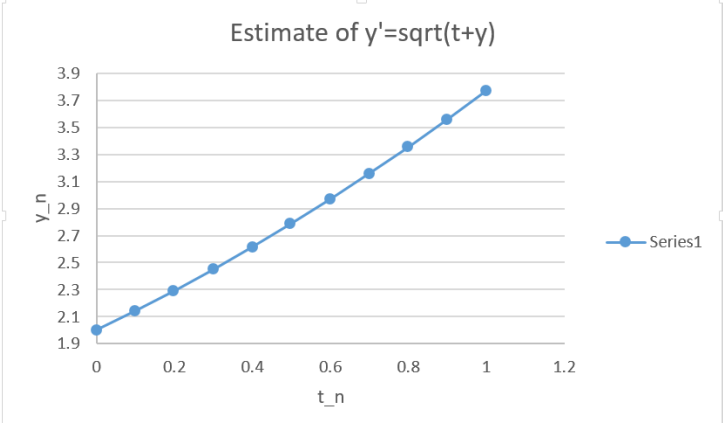
2. Your first task will be to complete the calculation of this question to estimate $y(2)$, usally all three methods. You can do this by copying the code from line three and copying them down until the value of $t_n = 2$. For instance, if we wanted to estimate the value of $y(1)$ using Euler's method, copy the code as shown on the screenshot below.

B16

	A	B	C	D	E	F	G
1	Step (n)	t_n	y_n	m_n=f(t_n,y_n)	Delta_t=h	y_(n+1)	
2	0	0	2	1.414213562	0.1	2.14142	
3	1	0.1	2.14142	1.497137721	0.1	2.29114	
4	2	0.2	2.29114	1.578333022	0.1	2.44897	
5	3	0.3	2.44897	1.658001336	0.1	2.61477	
6	4	0.4	2.61477	1.736308891	0.1	2.7884	
7	5	0.5	2.7884	1.813394456	0.1	2.96974	
8	6	0.6	2.96974	1.889375267	0.1	3.15868	
9	7	0.7	3.15868	1.964351401	0.1	3.35511	
10	8	0.8	3.35511	2.038409077	0.1	3.55895	
11	9	0.9	3.55895	2.111623184	0.1	3.77011	
12	10	1	3.77011	2.184059246	0.1	3.98852	
13							

Report the value of y_n at the START of that line as your estimate.

3. Once you've calculated your estimate, use the data in columns B and C to construct a graph. In Excel 2013, highlight the cells and then select a line graph from the list of recommended graphs. In earlier versions of Excel, the commands will be slightly different. A quick Google search for "line graph in Excel 2010" should provide several resources if the help feature isn't enough. An example graph for the first ten steps is shown below.



4. Once you've completed the estimate for $y(2)$ with the step size $h = 0.1$, change the step size to $h = 0.025$, and recalculate. Obviously, you will need more steps with a smaller step size. Preserve your work by taking a screenshot and saving it in a Word file, or copy the model code to a new worksheet in your Excel file. Repeat the steps above to calculate the estimate and produce a new graph. Be wary of how you paste graphs. The default option in more recent versions of Word links the graphs to the original data, so it will update if you change the Excel sheet. If you are reusing the same sheet for each calculation, be sure to select a paste option that unlinks the data.

5. Repeat these steps for each of the problems shown below with the indicated step size, to estimate y at the indicated point, each time preserving old work by creating a new sheet or taking a screenshot.
   a. $y' = 2 - t + y, h = 0.05, y(1)$
   b. $y' = 2 - t + y, h = 0.001, y(1)$
   c. $y' = \sin(y + t)\,, h = .1, y(3)$
   d. $y' = e^{-y^2+t}, h = 0.01, y(1)$

6. Answer the following questions.
   a. For the example problem(s) in question #4, and all the problems in question #5, paste your graphs, and your final estimate for the specified values.

   b. For the calculation in problem #4, which estimate for $y(2)$ do you think is better? Why? (Specify both the step size and the method.)

   c. For the calculations in #5a and #5b, compare your results to the exact value obtained by solving this linear question for the analytical solution. How do your estimates compare to the exact value? What is the magnitude of the error?

   d. Construct a graph that compares 5a, 5b, and your analytical solution on the same graph. [Hint: After solving for the exact solution for $y(t)$, create a column that solves for the value of y at each value of t_n using the formula you found for $y(t)$ and use that to construct your graph for the function.

   e. Of all your estimates, which do you think is likely to be the least accurate and why?

7. Remember, when you are finished, submit this sheet and your Excel file (a print-out is fine or submit by email).