MTH 267, Homework #9, Fall 2022 Name _____

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

1. Determine whether the function $f(t) = \begin{cases} t^2, & 0 \le t \le 1 \\ (t-1)^{-1}, & 1 < t \le 2 \\ 1, & 2 < t \le 3 \end{cases}$ is continuous or piecewise continuous on the interval [0,3].

2. Use the definition of the Laplace transform to find the transform $\mathcal{L}\{f(t)\}$of the following functions.
   a. $f(t) = t$
   b. $f(t) = \sinh bt$
   c. $f(t) = e^{at}\cos bt$ [Hint: You may be able to use your results from parts a-c for parts d-f.]
   d. $f(t) = t^2 \sinh bt$
   e. $f(t) = \cos at$
   f. $f(t) = te^{at}$
   g. $u_c(t) = \begin{cases} 0, t < c \\ 1, t \ge c \end{cases}$

3. Find the inverse Laplace transform $\mathcal{L}^{-1}\{F(s)\}$ of each of the following functions. Use the table of transforms. You will have to do partial fraction decomposition on some of these, or complete squares on others.
   a. $F(s) = \frac{3}{s^2+4}$
   b. $F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$
   c. $F(s) = \frac{3s}{s^2-s-6}$
   d. $F(s) = \frac{2s-3}{s^2+2s+10}$

4. For each of the rational expressions below, use partial fractions to rewrite the expression as a sum of expressions with a single linear factor or an unfactorable quadratic in each denominator. For all unfactorable quadratic denominators, rewrite the expression as the sum or difference of squares.
   a. $\frac{3x+2}{x^2+x}$
   b. $\frac{x^2+29x+5}{(x-4)^2(x^2+3)}$
   c. $\frac{3x+11}{x^2-x-6}$
   d. $\frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2}$
   e. $\frac{x^2+4}{3x^3+4x^2-4x}$
   f. $\frac{18}{x^3-3x^2}$