MTH 267, Homework #8, Fall 2022 Name _____

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

1. A spring with spring constant 4N/m is attached to a 1kg mass with negligible friction. If the mass is initially displaced to the right of equilibrium by 0.5m and has an initial velocity of 1 m/s toward equilibrium. Compute the amplitude of the oscillation.

2. A 16 pound weight is attached to a spring with friction constant 8lb · s/ft and spring constant 7lb/ft. Write the associated spring/mass ODE

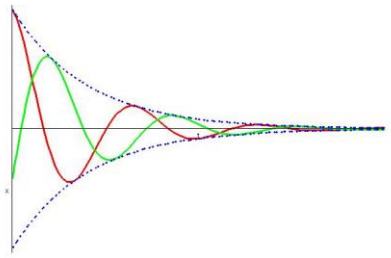
3. For the solution u(t), find the amplitude $R = \sqrt{A^2 + B^2}$, frequency ω and phase shift $\delta = \tan^{-1}\left(\frac{B}{A}\right)$, and use that information to write the solution as a single term of the form $u(t) = R\cos(\omega_0 t - \delta)$.
   a. $u(t) = 3\cos 2t + 4\sin 2t$ b. $u(t) = -2\cos \pi t - 3\sin \pi t$

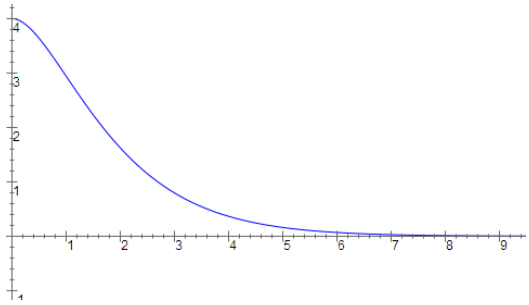
4. A series circuit has a capacitor of $C = 0.8 \times 10^{-6} F$ and an inductor of L=0.2 H, find the resistance R so that the circuit is critically damped.

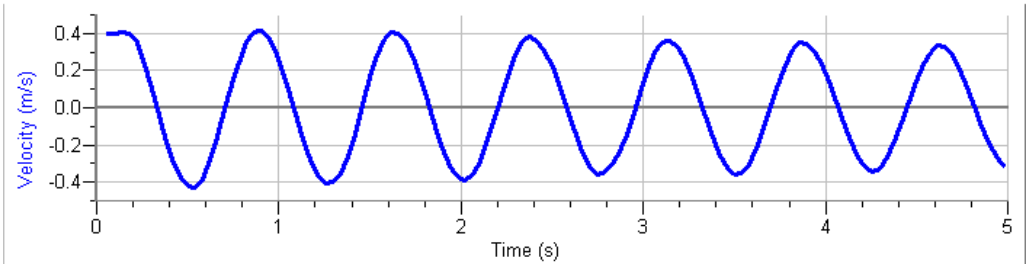
5. A series circuit has a capacitor of $10^{-5} F$, a resistor of $3 \times 10^{2} \Omega$ and an inductor of 0.2 H. The initial charge on the capacitor is $10^{-6} C$ and there is no initial current. Find the charge on the capacitor at any time t. Graph the solution.

6. A spring with spring constant 18N/m is attached to a 2kg mass with friction constant 4Ns/m. If the mass has initially position 1 meter to the right of equilibrium and has no initial velocity: (a) Find the solution, (b) Express the solution in phase/angle form, (c) Plot the solution together with its two bounding curves. Is the system undamped, underdamped, critically damped or overdamped?

7. For each of the graphs below, associate the graph with the characteristics of the system being modeled: undamped, underdamped, critically damped, overdamped.

a. 

c. 

b. 

8. Give an example of a problem whose solution has the following characteristics:
   a. Beats
   b. Resonance
   c. Asymptotically approaches zero
   d. Contains a transient solution
   e. Oscillating steady state solution
   f. No damping
   g. Critical damping

9. Plot the steady state solutions to $y'' + 2y' + 10y = 4$ and $y'' + 2y' + 10y = 4\,cos(2t)$. Does the associated system exhibit resonance?

10. Find the eigenvalues and eigenvectors of each matrix.
    a. $\begin{bmatrix} 1 & 6 \\ 2 & 5 \end{bmatrix}$
    b. $\begin{bmatrix} -1 & 1 \\ 3 & 1 \end{bmatrix}$
    c. $\begin{bmatrix} -4 & 1 \\ 6 & -5 \end{bmatrix}$
    d. $\begin{bmatrix} -2 & 2 \\ -5 & 6 \end{bmatrix}$
    e. $\begin{bmatrix} 2 & 9 \\ 1 & 10 \end{bmatrix}$
    f. $\begin{bmatrix} -2 & 5 \\ 7 & 0 \end{bmatrix}$
    g. $\begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$
    h. $\begin{bmatrix} -2 & 5 & 3 \\ 0 & 2 & -4 \\ 0 & -1 & 2 \end{bmatrix}$
    i. $\begin{bmatrix} 4 & 5 \\ 6 & 11 \end{bmatrix}$
    j. $\begin{bmatrix} -3 & 7 \\ 5 & -1 \end{bmatrix}$
    k. $\begin{bmatrix} -4 & 5 \\ -5 & -4 \end{bmatrix}$