MTH 267, Homework #7, Fall 2022 Name _____

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

1. Transform the given equations in a system of first order equations.
   a. $u'' + \frac{1}{2}u' + 2u = 0$
   b. $u^{IV} - u = 0$
   c. $t^2u'' + tu' + \left(t^2 - \frac{1}{4}\right)u = 0$
   d. $u'' + \frac{1}{4}u' + 4u = 2\cos 3t, u(0) = 1, u'(0) = -2$

2. Transform the given system into a single equation of higher order. Find the solution and sketch the graph.
   a. $x_1' = 3x_1 - 2x_2, x_1(0) = 3$
      $x_2' = 2x_1 - 2x_2, x_2(0) = \frac{1}{2}$
   b. $x_1' = 2x_2, x_1(0) = 3$
      $x_2' = -2x_1, x_2(0) = 4$

3. Use the method of reduction of order to find a second solution to the given differential equation.
   a. $t^2y'' + 2ty' - 2y = 0, t > 0, y_1(t) = t$
   b. $(x-1)y'' - xy' + y = 0, x > 1, y_1(x) = e^x$

1. Determine the intervals on which the solutions are sure to exist.
   c. $y^{IV} + 4y''' + 3y = t$
   $y''' + ty'' + t^2y' + t^3y = \ln t$

2. Verify that the given functions are solutions to the differential equation. Do they form a fundamental set? (Justify your answer.)
   a. $y''' + y' = 0, 1, \cos t, \sin t$
   b. $y^{IV} + 2y''' + y'' = 0, 1, t, e^{-t}, te^{-t}$
   c. $x^3y''' + x^2y'' - 2xy' + 2y = 0, x, x^2, \frac{1}{x}$
   d. $y^{IV} - y = 0, y(t) = c_1\cos t + c_2\sin t + c_3\cosh t + c_4\sinh t$

3. Use the method of reduction of order to solve the differential equation $(2-t)y''' + (2t-3)y'' - ty' + y = 0, t < 2, y_1(t) = e^t$ given one of the proposed solutions.

4. For each initial value problem, find the solution to the higher order differential equation. Plot the graph of the solution.
   a. $y''' + y' = 0, y(0) = 0, y'(0) = 1, y''(0) = 2$
   b. $y^{IV} - 4y''' + 4y'' = 0, y(1) = -1, y'(1) = 2, y''(1) = 0, y'''(1) = 0$
   c. $y^{IV} + 6y''' + 17y'' + 22y' + 14y = 0, y(0) = 1, y'(0) = -2, y''(0) = 0, y'''(0) = 3$
      [Hint: This polynomial factors as $(r^2 + 2r + 2)(r^2 + 4r + 7)$.]

5. Use the method of undetermined coefficients to solve the differential equations. Graph the solution.
   a. $y''' + 4y' = t, y(0) = y'(0) = 0, y''(0) = 1$
   b. $y^{IV} + 2y'' + y = 3t + 4, y(0) = y'(0) = 0, y''(0) = y'''(0) = 1$