MTH 267, Homework #6, Fall 2022   Name _____

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

1. Write the form of the proposed solution to the non-homogeneous equation by the method of undetermined coefficients. State the proposed form; you do not need to find the constants. You do need to be able to compare to the homogenous solutions, however.
   a. $y'' + 2y' + 2y = 3e^{-t} + 2e^{-t}\cos t + 4e^{-t}t^2 \sin t$
   b. $y'' + 4y = t^2 \sin 2t + (6t + 7)\cos 2t$

2. Find the solutions to the following initial value problems using the method of undetermined coefficients. Solve for all constants, then graph the equation.
   a. $2y'' + 3y' + y = t^2 + 3\sin t, y(0) = 0, y'(0) = 1$
   b. $y'' + y' + 4y = 2\sinh t, y(0) = 1, y'(0) = 0$

3. Use the method of variation of parameters to find the solution to the differential equation.
   a. $y'' + y = \tan t, 0 < t < \frac{\pi}{2}$ d. $y'' - 2y' + y = \frac{e^t}{1+t^2}$
   b. $ty'' - (1 + t)y' + y = t^2 e^{2t}, t > 0, y_1(t) = 1 + t, y_2(t) = e^t$
   c. $x^2 y'' - 3xy' + 4y = x^2 \ln x, x > 0, y_1(t) = x^2, y_2(t) = x^2 \ln x$

4. For a second order differential equation $ay'' + by' + cy = g(t)$, for the function g(t), list 5 functions you can use the method of undetermined coefficients on; list 5 functions you must use variation of parameters on.