MTH 267, Homework #5, Fall 2022 Name _____

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

1. Solve the following equations for all real and complex solutions. Factor where possible, and then apply the quadratic formula or other techniques where needed.
   a. $x^2 + 3x - 4 = 0$
   b. $18x^3 - 63x^2 + 9x = 0$
   c. $125 - 8y^3 = 0$
   d. $64r^3 + 27 = 0$
   e. $2m^3 - 18m = 0$
   f. $2x^2 - 98 = 0$
   g. $3r^2 - 75 = 0$
   h. $x^2 + 11x + 24 = 0$
   i. $14t^2 - 9t + 1 = 0$
   j. $n^3 - 28 + 7n^2 - 4n = 0$
   k. $20 - 3x - 2x^2 = 0$
   l. $16a^2 - 56a + 49 = 0$
   m. $12x^2 + 34x + 24 = 0$
   n. $x^4 - 14x^2 - 32 = 0$
   o. $s^6 - 1 = 0$

2. Find the Wronskian of the following sets of solutions. Do the solutions form a fundamental set?
   a. $\cos t\,, \sin t$
   b. $t^2, t^2 \ln t\,, t^{-4}$
   c. x, xe^x
   d. $\sinh t\,, \cosh t\,, e^t$
   e. $e^t \sin t\,, e^t \cos t$

3. Use Abel's Theorem and the given initial values to determine the longest interval on which the solution is guaranteed to exist. [Do not solve the equations.]
   a. $ty'' + 3y = t, y(1) = 1, y'(1) = 2$
   b. $t(t-4)y'' - 3ty' + 4y = 2, y(3) = 0, y'(3) = -1$
   c. $x^2(x^2-9)y'' - xy' + y = 0, y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = 0$

4. Use Euler's formula to write the following expressions in the form a+bi.
   a. e^{1+2i}
   b. 2^{1-i}
   c. $e^{2-\frac{\pi}{2}i}$

5. Express the given complex number in the form $R(\cos\theta + i\sin\theta) = Re^{i\theta}$.
   a. $1 - i$
   b. $1 + 2i$
   c. $\sqrt{3} - i$
   d. $-2i$

6. Solve the following Cauchy-Euler equations.
   a. $x^2y'' + xy' + y = 0$
   b. $t^2y'' + 5ty' + 13y = 0$
   c. $t^2y'' - ty' + 5y = 0$

7. Find the solutions to the following initial value problems. Find any critical points of the solution if they exist. Does the function approach a limit asymptotically?
   a. $6y'' - 5y' + y = 0, y(0) = 4, y'(0) = 0$
   b. $2y'' - 3y' + y = 0, y(0) = 2, y'(0) = \frac{1}{2}$
   c. $y'' + 4y' + 5y = 0, y(0) = 1, y'(0) = 0$
   d. $y'' + 4y' + 4y = 0, y(-1) = 2, y'(-1) = 1$