MTH 267, Homework #3, Fall 2022   Name _____

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

1. Prove that the differential equation is homogeneous. State the degree. Then solve the equation using an appropriate substitution.
   a. $(x - y)dx + xdy = 0$
   b. $\frac{dy}{dx} = \frac{x+3y}{3x+y}$
   c. $ydx + x(lnx - lny - 1)dy = 0, y(1) = e$

   d. $(y^2 + yx)dx - x^2 dy = 0$
   e. $y' = \frac{xy}{x^2 - y^2}$

2. Solve the following linear equations using the method of integrating factors. Find all missing constants if an initial value is provided.
   a. $y' - 2y = 3e^t$
   b. $ty' - y = t^2 e^{-t}, t > 0$
   c. $t^3 y' + 4t^2 y = e^{-t}, y(-1) = 0, t < 0$

   d. $ty' + 2y = \sin t, t > 0$
   e. $y' - 2y = e^{2t}, y(0) = 2$

3. The method of variation of parameters for (first order) linear equations is derived from the method of integrating factors, and produces the solution to a linear equation of the form $y' + p(t)y = g(t)$ as $y = \frac{1}{\mu}[\int \mu\, g(t)dt + C]$, where $\mu = e^{\int p(t)dt}$. Redo the problems in question #1 by this method.

4. Consider a tank with 200 liters of salt-water solution, 30 grams of which is salt. Pouring into the tank is a brine solution at a rate of 4 liters/minute and with a concentration of 1 grams per liter. The "well-mixed" solution pours out at a rate of 5 liters/minute. Find the amount at time t.

5. Solve the Bernoulli equation $t^2 y' + 2ty - y^3 = 0, t > 0$.

6. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 200° when freshly poured, and 1 minute later cooled to 190° in a room at 70°, determine when the coffee reaches a temperature of 150° (in minutes).

7. A certain college graduate borrows $8000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate k, determine the payment rate k that is required to pay off the loan in 3 years. Also determine how much interest is paid during the three-year period.