

4. Solve the homogeneous second order ODE $y'' + 4y' + 3y = 0$, $y(0) = 2$, $y'(0) = -1$. (12 points)

5. Solve the differential equation $2y^{IV} + 13y''' - 5y'' + 13y' - 7y = 0$ (15 points)

6. A mass weighing 64 lbs stretches a spring 9 in. The mass is attached to a viscous damper with a damping constant of 6 lbs·sec/ft.

a. If the mass is set in motion from its equilibrium position with a downward velocity of 1 in/sec, find its position y at any time t . (16 points)

b. Sketch the graph of the function. (5 points)

- c. Is it undamped, underdamped, critically damped or overdamped? How can you tell? (5 points)

 - d. Does the system experience resonance, beats or neither? (5 points)

 - e. What is the transient solution? (3 points)

 - f. What is the steady state solution? (3 points)
7. Use the **definition** of Laplace transforms to find the transform $F(s)$ for the function $f(t) = te^t$. You may check your answers with the included table, but you must show the integration work to receive credit. (15 points)

8. Find the Laplace Transforms for the following functions. You may use the table of formulas. (5 points each)

a. $\mathcal{L}\{3 \cos(7t)\}$

b. $\mathcal{L}\left\{-\frac{1}{2}t^4 e^{-2t}\right\}$

c. $\mathcal{L}\{6 \sinh(\pi t)\}$

d. $\mathcal{L}\{3t \cosh(t)\}$

e. $\mathcal{L}\{4u_3(t) - 2u_5(t)\}$

9. Find the inverse Laplace transform of the following functions. (5 points each)

a. $\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2+4}\right\}$

b. $\mathcal{L}^{-1}\left\{\frac{4s}{s^2-6s+7}\right\}$

c. $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2}\right\}$

d. $\mathcal{L}^{-1}\left\{\frac{3s-7}{s^2+4s+8}\right\}$

e. $\mathcal{L}^{-1}\left\{\frac{10}{(s-4)^7}\right\}$

10. Solve the differential equations using Laplace Transforms. (15 points each)

a. $y'' + 6y' - 7y = te^t, y(0) = 0, y'(0) = 2$

b. $y'' - 5y' + 6y = f(t), y(0) = 1, y'(0) = 0$, for $f(t) = \begin{cases} 0, & 0 \leq t < 3 \\ 4, & t \geq 3 \end{cases}$

11. Find the inverse Laplace transform of $F(s) = \frac{5}{s^4 + 49s^2}$ using convolutions. (10 points)

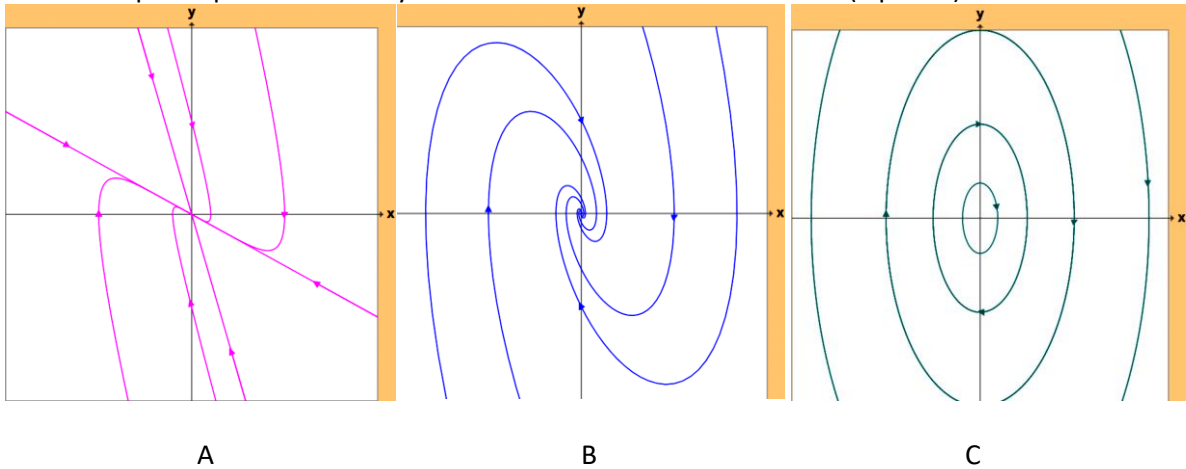
12. Solve the systems of equations for the general solution below using eigenvalues. Be sure that your solutions are expressed only with real-valued functions. (14 points each)

a. $\vec{x}'(t) = \begin{pmatrix} 3 & 2 \\ 3 & 4 \end{pmatrix} \vec{x}$

b. $\vec{x}'(t) = \begin{pmatrix} -8 & 9 \\ -5 & 4 \end{pmatrix} \vec{x}$

13. Solve the differential equation $\frac{dy}{dt} = \frac{t\sqrt{1-y^2}}{e^{2t}}$ using separation of variables. (15 points)

14. A series of phase portraits for a system of linear ODEs is shown below. (9 points)



Match the phase portrait with one of the differential equations below that could be represented by the graph. Explain your reasoning.

a. $y'' + 3y = 0$

b. $y'' + 4y' + 3y = 0$

c. $y'' + 2y' + 5y = 0$

15. Use the Existence and Uniqueness Theorem to determine where the differential equation

$y' = \frac{2\sqrt{1-y^2}}{x^2}$ is guaranteed to have a unique solution. Sketch the graph of the region. (12 points)

$$\text{Modified Euler: } y_{n+1} = y_n + hf \left[t_n + \frac{1}{2}h, y_n + \frac{1}{2}hf(t_n, y_n) \right]$$

$$\text{Runge-Kutta: } y_{n+1} = y_n + h \left(\frac{k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4}}{6} \right),$$

$$k_{n1} = f(t_n, y_n), k_{n2} = f \left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_{n1} \right),$$

$$k_{n3} = f \left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_{n2} \right), k_{n4} = f(t_n + h, y_n + hk_{n3})$$