MTH 264, Quiz #5, Fall 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the number of steps needed to achieve an error $E \le 10^{-7}$ for the integral $\int \ln x dx$ over the interval [1,4] using Simpson's Rule. You may use the error formula $E \le \frac{(b-a)^5}{180n^4}\left[\max\left|f^{IV}(x)\right|\right]$.

$y = \ln x$

$y' = \frac{1}{x} = x^{-1}$

$y'' = -x^{-2}$

$y''' = +2x^{-3}$

$y^{IV} = -6x^{-4} = \frac{-6}{x^4}$

max when x is smallest (in denominator) so $\max|f^{IV}(x)|$ on $[1,4]$ is $\left|\frac{-6}{(1)^4}\right| = 6$

$$E \le \frac{(4-1)^5}{180n^4}(6) = \frac{6(243)}{180n^4}$$

$$n^4 \ge \frac{6(243)}{180(10^{-7})} = 81 \times 10^6$$

$$n \ge 94.8683$$

$n = 96$ (must be even)

2. Use the integration tables to integrate the following integrals.

$\int \sqrt{x}\tan^{-1} x^{3/2} dx$ recall that $\tan^{-1}(x) = \arctan(x)$

$u = x^{3/2}$

$du = \frac{3}{2}x^{1/2}dx$

$\frac{2}{3}du = \sqrt{x}\,dx$

$$\frac{2}{3}\int \tan^{-1} u\, du$$

$$\frac{2}{3}\left[u \arctan u - \frac{1}{2}\ln|u^2+1|\right] + C$$

$$\frac{2}{3}x^{3/2}\arctan x^{3/2} - \frac{1}{3}\ln|x^3+1| + C$$

3. Use the Trapezoidal Rule to approximate the integral $\int_0^2 x^3 dx$ with n=4.

$$h = \frac{2-0}{4} = \frac{1}{2} \qquad \{0, \tfrac{1}{2}, 1, \tfrac{3}{2}, 2\}$$

$$\int_0^2 x^3 dx \approx \frac{1}{2}\cdot\frac{1}{2}\left[f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)\right] =$$

$$\frac{1}{4}\left[0^3 + 2(\tfrac{1}{2})^3 + 2(1)^3 + 2(\tfrac{3}{2})^3 + 2^3\right] =$$

$$\frac{1}{4}\left[0 + \frac{1}{4} + 2 + \frac{27}{4} + 8\right) = \frac{17}{4} = 4.25$$