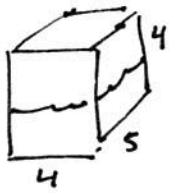
MTH 264, Quiz #3, Fall 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. A rectangular tank with a base 4 feet by 5 feet and a height of 4 feet is full of water. The water weighs 62.4 pounds per cubic foot. How much work is done in pumping water out over the top edge in order to empty the entire tank?



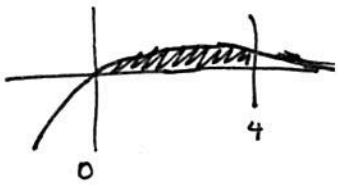
$A = 20\,dy$

$\int_0^4 62.4 \cdot 20(4-y)\,dy = 20 \cdot 62.4\left[4y - y^2/2\right]_0^4 =$

$20(62.4)(8) = 9984$ ft lbs.

2. Find the centroid of the region bounded by the graphs of $y = xe^{-x/2}$, $y = 0$, $x = 0$ and $x = 4$.

you can use technology to do the integration



$M = \rho\int_0^4 xe^{-x/2}\,dx = 4 - 12e^{-2} \approx 2.37598$

$M_x = \frac{\rho}{2}\int_0^4 (xe^{-x/2})^2\,dx = \frac{1}{2}\int_0^4 x^2e^{-x}\,dx =$

$(e^4 - 13)e^{-4} \approx 0.761897$

$M_y = \rho\int_0^4 x \cdot xe^{-x/2}\,dx = \int_0^4 x^2e^{-x/2}\,dx =$

$16 - 80e^{-2} \approx 5.17318$

$\bar{x} = 2.17728 \quad \bar{y} = 0.320667$

3. Integrate.

a. $\int \frac{\operatorname{csch}\left(\frac{1}{\gamma}\right)\coth\left(\frac{1}{\gamma}\right)}{\gamma^2}\,d\gamma$

$u = 1/\gamma \quad du = -\frac{1}{\gamma^2}\,d\gamma$

$\int -\operatorname{csch} u \coth u\,du = \operatorname{csch} u + C$

$= \operatorname{csch}\left(\frac{1}{\gamma}\right) + C$

b. $\int \frac{2\rho}{\rho - 4}\,d\rho$

$$\begin{array}{r|l} & 2 \\ \hline \rho - 4 & 2\rho + 0 \\ & -2\rho + 8 \\ \hline & 8 \end{array}$$

$\int 2 + \frac{8}{\rho - 4}\,d\rho =$

$2\rho + 8\ln|\rho - 4| + C$