10/18/2022

Finish Series Tests
Power Series

Write $0.\overline{46}$ as a series, and use that to write the equivalent fraction.

0.464646464646…
0.46 = $\frac{46}{100} = 46\left(\frac{1}{10^2}\right)$
0.0046 =$\frac{46}{10{,}000} = 46\left(\frac{1}{10^4}\right)$
0.000046 = $\frac{46}{10^6} = 46\left(\frac{1}{10^6}\right)$

Add these together (and continue the pattern) we can get 0.464646…

$$r = \frac{1}{10^2}$$

We might think that a=46, but that only works if we start with n=1

$$\sum_{n=0}^{\infty} 46\left(\frac{1}{10^2}\right)^n$$

My first number would not be 0.46, it would 46.

$$\sum_{n=0}^{\infty} \frac{46}{100}\left(\frac{1}{10^2}\right)^n$$

This will work for our geometric series.

$$Sum = \frac{a}{1-r} = \frac{\frac{46}{100}}{1-\frac{1}{100}} = \frac{\frac{46}{100}}{\frac{99}{100}} = \frac{46}{100} \times \frac{100}{99} = \frac{46}{99}$$

See Series Practice handout for the discussion.

Power Series.

Convergence or divergence of power series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{n^3}$$

We want to know for which values of x does the series converge.

Usually, the best bet for determining convergence is the ratio test.

Recall the ratio test compares a_n term with a_{n+1} and if the limit is less than 1, the series converges, and if it is greater than 1, it diverges; and if it is equal to 1, the test is indeterminant.

$$\lim_{n\to\infty} \frac{(x+1)^{n+1}}{(n+1)^3} \times \frac{n^3}{(x+1)^n} = \lim_{n\to\infty} \frac{\cancel{(x+1)^n}(x+1)}{\cancel{(x+1)^n}} \times \cancelto{1}{\frac{n^3}{(n+1)^3}} = \lim_{n\to\infty} |x+1| < 1$$

$$-1 < x+1 < 1$$

$$-2 < x < 0$$

The interval of convergence is the values of x for which the series converges. So far: $(-2,0)$.

The radius of convergence is 1. If my interval is (a,b), then the radius of convergence is $R = \frac{b-a}{2}$

To find the full interval of convergence, we have to check the endpoints: does the series converge at -2 or at 0.

When $x = -2$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-2+1)^n}{n^3} = \sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{n^3} = \sum_{n=0}^{\infty} \frac{(-1\times -1)^n}{n^3} = \sum_{n=0}^{\infty} \frac{1}{n^3}$$

By the p-series, this series converges at $x = -2$.

When $x = 0$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(0+1)^n}{n^3} = \sum_{n=0}^{\infty} \frac{(-1)^n(1)^n}{n^3} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^3}$$

This converges by the alternating series at $x = 0$

So the full interval of convergences is $[-2,0]$ or , $-2 \le x \le 0$

General procedure:
Start with ratio test (usually)
Find general conditions on an open interval for x
If non-infinite, then also test the endpoints to see if one, both or neither are included.