MTH 324, Homework #3, Fall 2022 Name ____

Instructions: Show work or attach R code used to perform calculations (or any other technology used). Be sure to answer all parts of each problem as completely as possible, and attach work to this cover sheet with a staple.

1. Let X be the time between two successive arrivals at the drive-up window of a local bank. If X has an exponential distribution with $\lambda = 1$, compute the following.
   a. The expected time between two successive arrivals.
   b. The standard deviation of the time between successive arrivals.
   c. $P(X \leq 4)$
   d. $P(2 \leq X \leq 5)$

2. Evaluate the following.
   a. $\Gamma(6)$
   b. $\Gamma\left(\frac{5}{2}\right)$

3. A certain market has both an express checkout line and a super-express checkout line. Let X_1 denote the number of customers in line at the express checkout at particular time of the day, and let X_2 denote the number of customers in line at the super-express checkout at the same time. Suppose the joint probability mass function is given in the table below.

		x_2			
		0	1	2	3
	0	0.08	0.07	0.04	0.00
x_1	1	0.06	0.15	0.05	0.04
	2	0.05	0.04	0.10	0.06
	3	0.00	0.03	0.04	0.07
	4	0.00	0.01	0.05	0.06

   a. What is $P(X_1 = 1, X_2 = 1)$, that is, the probability that exactly one person is in each lines.
   b. What is $P(X_1 = X_2)$, that is the probability that the numbers of customers in the two lines are identical?
   c. Let A denote the event that there are at least two more customers in one line than in the other line. Express A in terms of X_1 and X_2, and calculate the probability of this event.
   d. Find P_{X_1}, P_{X_2}.
   e. What is $P(X_1 + X_2 \leq 1)$?
   f. What is $P(X_1 + X_2 = 4)$?
   g. Find $E(X_1), E(X_2), E(X_1X_2)$.
   h. What is $E(X_1 + X_2)$?
   i. Find $V(X_1), V(X_2), Cov(X_1, X_2)$.
   j. Are X_1, X_2 independent? Why or why not?

4. Find all first partial derivatives.
   a. $f(x, y) = x^2 - 3y^2 + 7$
   b. $z = \dfrac{xy}{x^2 + y^2}$
   c. $f(x, y) = x^2 + 4xy + y^2 - 4x + 16y + 3$

d. $f(x,y,z)=\dfrac{3xz}{x+y}$

e. $w=3x^2y-5xyz+10yz^2$

5. Evaluate the integral. Sketch or describe the region

a. $\int_0^1\int_0^2 (x+y)dydx$

c. $\int_1^4\int_1^{\sqrt{x}} 2ye^{-x}dydx$

b. $\int_0^2\int_{3y^2-6y}^{2y-y^2} 3ydxdy$

d. $\int_{-1}^1\int_{-1}^1\int_{-1}^1 x^2y^2z^2dxdydz$

6. The joint density function for a pair of random variables is given by the functions below. Find the value of C in each case, and then use it to find the indicated probabilities.

a. $f(x,y)=\begin{cases} Cx(1+y), & 0\le x\le 1,\ \ 0\le y\le 2 \\ 0, & otherwise \end{cases}, P\left(X\ge\frac{1}{2}\right), P\left(X\ge\frac{1}{2}, Y\le\frac{1}{2}\right), P(X+Y\le 1)$

b. $f(x,y)=\begin{cases} Ce^{-\left(\frac{1}{2}x+\frac{1}{5}y\right)}, & x\ge 0,\ \ y\ge 0 \\ 0, & otherwise \end{cases}, P(Y\ge 1), P(X\le 2, Y\le 4)$

7. Each front tire on a particular type of vehicle is supposed to be filled to a pressure of 26 psi. Suppose the actual air pressure in each tire is a random variable—X for the right tire and Y for the left tire, with a joint probability density function

$$f(x,y)=\begin{cases} K(x^2+y^2), 20\le x\le 30, 20\le y\le 30 \\ 0, \qquad otherwise \end{cases}$$

a. What is the value of K?
b. What is the probability that both tires are underfilled?
c. What is the probability that the difference in air pressure between the two tires is at most 2 psi?
d. Determine the marginal distribution of air pressure in the right tire alone.
e. Are X and Y independent random variables?

8. Show that if $Y=aX+b\ (a\neq 0)$, then $Corr(X,Y)=\pm 1$. Under what conditions will $\rho=+1$?

9. Determine the values of the following quantities.

a. $\chi^2_{0.1,15}$ b. $\chi^2_{0.1,25}$ c. $\chi^2_{0.01,25}$ d. $\chi^2_{0.005,25}$ e. $\chi^2_{0.99,25}$ f. $\chi^2_{0.995,25}$

10. Determine

a. The 95th percentile of the chi-squared distribution with $\nu=10$.
b. The 5th percentile of the chi-squared distribution with $\nu=10$.
c. $P(10.98\le\chi^2\le 36.78)$ where χ^2 is a chi-squared random variable with $\nu=22$.
d. $P(\chi^2<14.611\ or\ \chi^2>37652)$ where χ^2 is a chi-squared random variable with $\nu=25$.