Math 266, Quiz #7, Fall 2021 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the nullspace of the matrix $A = \begin{bmatrix} 1 & 3 & -4 & -3 & 1 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

finish reducing $\rightarrow \begin{bmatrix} 1 & 0 & 5 & -6 & 1 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$\begin{aligned} x_1 + 5x_3 - 6x_4 + x_5 &= 0 \\ x_2 - 3x_3 + x_4 &= 0 \\ x_3 &= x_3 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned}$$

$$\begin{aligned} x_1 &= -5x_3 + 6x_4 - x_5 \\ x_2 &= 3x_3 - x_4 \\ x_3 &= x_3 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned} \quad \rightarrow \vec{x} = \begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 6 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_5$$

2. Determine if the following sets of vectors are linearly independent. Then determine if they form a basis for the specified space. Explain your reasoning.

a. $\left\{ \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}, R^3$

independent yes
do not form a basis for $\mathbb{R}^3$. need at least 3 vectors for that.

b. $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}, R^2$

dependent.
do not form a basis for $\mathbb{R}^2$ - too many vectors

c. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 3 \\ 3 \end{bmatrix} \right\}, R^4$

rref $\Rightarrow$ $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

dependent
not a basis for $\mathbb{R}^4$