

$$\begin{bmatrix} 2 & 3 & -1 \\ 6 & 5 & 0 \\ 2 & -5 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix}$$

1.

$$\begin{bmatrix} 2 & 3 & -1 \\ 6 & 5 & 0 \\ 2 & -5 & 7 \end{bmatrix}$$

Step 1. Make a pivot in a_{11}

$$\frac{1}{2}R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} \\ 6 & 5 & 0 \\ 2 & -5 & 7 \end{bmatrix}$$

Step 2. Eliminate everything below the pivot.

Multiple the pivot by the negative of the coefficient in the row to be worked on

$$-6R_1 + R_2 \rightarrow R_2$$

Add off to the side

$$\begin{array}{ccc} -6 & -9 & 3 \\ 6 & 5 & 0 \end{array}$$

Replace row 2, don't change anything else.

$$\begin{array}{ccc} 0 & -4 & 3 \end{array}$$

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} \\ 0 & -4 & 3 \\ 2 & -5 & 7 \end{bmatrix}$$

$$-2R_1 + R_3 \rightarrow R_3$$

$$\begin{array}{ccc} -2 & -3 & 1 \\ 2 & -5 & 7 \end{array}$$

$$\begin{array}{ccc} 0 & -8 & 8 \end{array}$$

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} \\ 0 & -4 & 3 \\ 0 & -8 & 8 \end{bmatrix}$$

When the column under the pivot is all zero, then make a new pivot in the next column.

Repeat cycle.

$$-\frac{1}{4}R_2 \rightarrow R_2$$

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{3}{4} \\ 0 & -8 & 8 \end{bmatrix}$$

$$8R_2 + R_3 \rightarrow R_3$$

$$\begin{bmatrix} 0 & 8 & -6 \\ 0 & -8 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 2 \end{bmatrix}$$

Multiply the pivot row, add to the row to be changed, put back in the non-pivot row used.

$$\frac{1}{2}R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix}$$

2.

$$\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{2}R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix}$$

$$-2R_1 + R_3 \rightarrow R_3$$

$$\begin{bmatrix} -2 & -4 & -4 & -4 \\ 2 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & -4 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -4 & -4 \end{bmatrix}$$

$$R_2 + R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -6 & -6 \end{bmatrix}$$

$$-\frac{1}{6}R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

This is echelon form. Now reduced echelon form.

Make coefficients about the pivot into zeros

$$2R_3 + R_2 \rightarrow R_2$$

$$\begin{array}{cccc} 0 & 0 & 2 & 2 \\ 0 & 1 & -2 & -2 \end{array}$$

$$0 \quad 1 \quad 0 \quad 0$$

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$-2R_3 + R_1 \rightarrow R_1$$

$$\begin{array}{cccc} 0 & 0 & -2 & -2 \\ 1 & 2 & 2 & 2 \end{array}$$

$$1 \quad 2 \quad 0 \quad 0$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$-2R_2 + R_1 \rightarrow R_1$$

$$\begin{array}{cccc} 0 & -2 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{array}$$

$$1 \quad 0 \quad 0 \quad 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

3.

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ -2 & 1 & 6 & 7 \\ 3 & -2 & -5 & -3 \end{bmatrix}$$

$$2R_1 + R_2 \rightarrow R_2$$

$$\begin{array}{cccc} 2 & 0 & -4 & -2 \\ -2 & 1 & 6 & 7 \end{array}$$

$$0 \quad 1 \quad 2 \quad 5$$

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 3 & -2 & -5 & -3 \end{bmatrix}$$

$$-3R_1 + R_3 \rightarrow R_3$$

$$\begin{array}{cccc} -3 & 0 & 6 & 3 \\ 3 & -2 & -5 & -3 \end{array}$$

$$0 \quad -2 \quad 1 \quad 0$$

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$$2R_2 + R_3 \rightarrow R_3$$

$$\begin{array}{cccc} 0 & 2 & 4 & 10 \\ 0 & -2 & 1 & 0 \end{array}$$

$$0 \quad 0 \quad 5 \quad 10$$

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

$$\frac{1}{5}R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$-2R_3 + R_2 \rightarrow R_2$$

$$2R_3 + R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

4.

$$\begin{bmatrix} 0 & 1 & 4 & 0 \\ 1 & 2 & -1 & 0 \\ 5 & 8 & 0 & 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 5 & 8 & 0 & 0 \end{bmatrix}$$

$$-5R_1 + R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & -2 & 5 & 0 \end{bmatrix}$$

$$2R_2 + R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 13 & 0 \end{bmatrix}$$

$$\frac{1}{13}R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$-4R_3 + R_2 \rightarrow R_2$$

$$R_3 + R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$-2R_2 + R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

