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Chapter 5: Introduction to Algebra

Terms with variables, or expressions with variables.

Default variable is usually x, the symbol stands in place of a number that is unknown
Algebra is fundamentally a series of logical steps based on the properties of numbers.

Expressions do not have equal signs whereas equations do.

Expression $x^2 + 3, y^3, \frac{4x^4}{5y^7}$

Equation $x + 3 = 2x - 5$

Term: a term in an expression or equation is an algebraic expression that is separated from other terms by a + or – sign.

$x^2 + 3$ there are two terms.

$\frac{4x^4}{5y^7}$ is only one term.

Polynomials: an expression in which every terms contains a variable to a power that is a non-negative integer (a whole number: 0, 1, 2, 3, 4…)

The power of 0 allows us to include constants without variables.
Multiples of 1, multiples of x, multiples of x^2, multiples of x^3, etc.

$$x^2 + 3x + 4$$

If there is one term in the expression: monomial
If there are two terms in the expression: binomial
If there are three terms in the expression: trinomial
If there are four or more terms in the expression: no special name: polynomials

Properties of numbers:

Order of operations: parentheses, exponents, multiplication & division, addition & subtraction

Like terms: are expressions where the variable and its power are identical and the expressions differ from each other only by a constant multiple

Like terms: $3x$ and $15x$, or $2x^2$ and $-x^2$, or 15 and 23

Unlike terms: $3x$ and $15y$, $2x^2$ and $-x$, $15xy$ and $23x^2y$, 3 and $18x$

Simplifying expressions are going to be to remove parentheses, and combine like terms.

Distributive rule: $a(b + c) = ab + ac$

$$3(x + 4) = 3x + 12$$

$$-(x + 4) = -x - 4$$

Evaluating expressions: value of the variable is supplied, and you need to replace the variable with the value and simplify.

$$x = 2, y = 3$$

$$x^2 + 3xy - y^2$$

$$(2)^2 + 3(2)(3) - (3)^2 = 4 + 18 - 9 = 13$$

$$x = -1, y = 2$$

$$x^2 + 3xy - y^2$$

$$(-1)^2 + 3(-1)(2) - (2)^2 = 1 - 6 - 4 = -9$$

When making a substitution of your own to check your work, don't generally use 0 or 1

Can test in the original expression and in the algebraically reduced expression to see if they produce the same result.

Expressions are usually organized in descending or ascending order
Descending: start with highest power first and end with the constant
Alphabetical ordering: alphabetize variables

Simplifying expressions (addition and subtraction of polynomials)

$$(6x - 7y + 11) + (2x + 9y + 12)$$
$$6x - 7y + 11 + 2x + 9y + 12$$
$$(6x + 2x) + (-7y + 9y) + (11 + 12)$$
$$(6 + 2)x + (-7 + 9)y + (23)$$
$$8x + 2y + 23$$

$$(2x + 9y + 12) - (6x - 7y + 11)$$
$$2x + 9y + 12 - 6x + 7y - 11$$
$$(2x - 6x) + (9y + 7y) + (12 - 11)$$
$$-4x + 16y + 1$$

The numbers in front of a variable are called coefficients. The number standing alone with no variable is called a constant.

$$(3x + 4) - (2x - 5) - (6x + 7)$$

Be careful of multiple minus signs: only apply to the () it's sitting in front of

Distributive rule applies for subtraction and also multiply by a constant or another variable.

$$2(x + 5) - 3(2x - 7)$$

$$2x + 10 - 6x + 21$$
$$-4x + 31$$

When adding expressions vertically (stacked on top of each other instead of side-by-side), it's customary to line up the common terms.

Multiplying expressions with variables (monomials) (if you remember FOIL, we're not doing that).

$$3x^4 \times 4x^3$$

Multiply the coefficients, apply exponent rules to multiple common variables.

$$(3 \times 4) \cdot (x^4 \cdot x^3) = 12x^7$$

$$3xy \cdot 4x^2y$$

$$(3 \cdot 4) \cdot (x \cdot x^2) \cdot (y \cdot y) = 12x^3y^2$$

A polynomial has a degree that is based on the "power" of the highest degree term.
Degree of a monomial in one variable: look to the power of the variable.

$12x^7$ is a monomial of degree 7

$$12x^7 + 10x^3 + 99x$$

This is also a polynomial of degree 7

If there are multiple variables, you have to add the powers of the variable to get the degree

$12x^3y^2$ is degree 5 (3+2)

$$12x^5, 12x^4y, x^3y^2, x^2y^3, xy^4, y^5$$

$$3x^2(9x - 5y + 2)$$

$$(3x^2 \cdot 9x) - (3x^2 \cdot 5y) + (3x^2 \cdot 2)$$

$$27x^3 - 15x^2y + 6x^2$$

$$\frac{2}{3}x^4 \cdot \frac{4}{5}x^3 = \frac{8}{15}x^7$$

Raising monomials to a power

$$(-2x^2)^3$$
$$(-2)^3(x^2)^3 = -8x^6$$

$$2x^2(3x^3)^4$$

Do the exponent portion (4th power) first, then multiply the result by $2x^2$.

Negatives: inside the parentheses behave differently than outside the parentheses.

$$-(-2x^2)^3 = -(-2)^3(x^2)^3 = -(-8x^6) = 8x^6$$

Different than $-(-8 + x^6)$

Division of monomials

$$\frac{81x^4}{18x^3} = \frac{9x}{2}$$

Divide the constants, subtract powers of the common variable

$$\frac{54x^2y^5}{18xy^7} = \frac{3x}{y^2}$$

If you are dividing a polynomial by a monomial, separate into terms and then simplify each term.

$$\frac{16x^4 - 12x^3 + 6x}{2x^2} = \frac{16x^4}{2x^2} - \frac{12x^3}{2x^2} + \frac{6x}{2x^2} = 8x^2 - 6x + \frac{3}{x}$$

We are not going to cover anything more complicated than this: no division with binomials or larger.

Problems from 5.1, 5.2, 5.4, 5.6