12/4/2020
Chapter 14

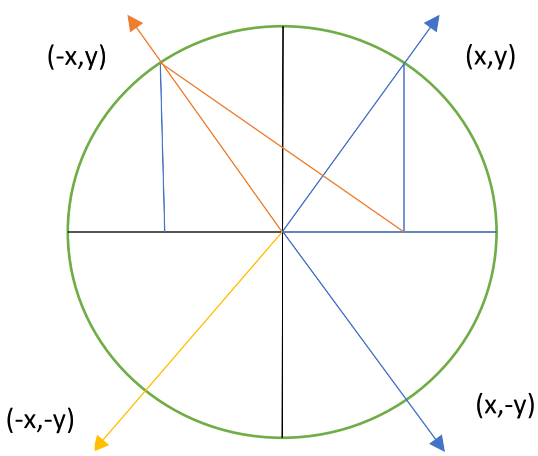
Extend the definitions of sine and cosine functions to angles that are not acute: 1) special angles like 0, 90-degrees, and obtuse angles (to any angle, even more than 180-degrees, or more than 360-degrees)

$$\sin(x)$$

$\sin(x)$	0.2588..	0.1736…	0.01745..	0.0001745…	0.000001745..	0
x	15-degrees	10-degrees	1-degree	0.01	0.0001	0
$\cos(x)$	0.9659…	0.9848…	0.9998…	0.9999999848		1

$\sin(x)$	0.0.9659	0.9848…	0.9998…	0.9999999848		1
x	75-degrees	80-degrees	89-degree	89.99	89.9999	90
$\cos(x)$	0.2588..	0.1736…	0.01745..	0.0001745…		0

Special angles: 0, 90, 180, 270, 360…

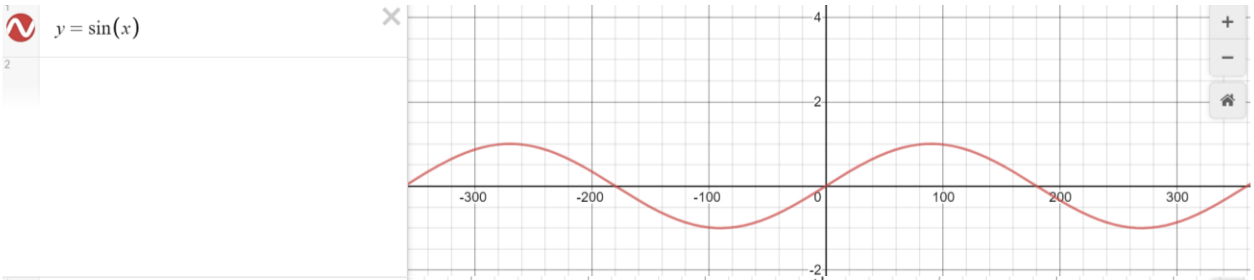


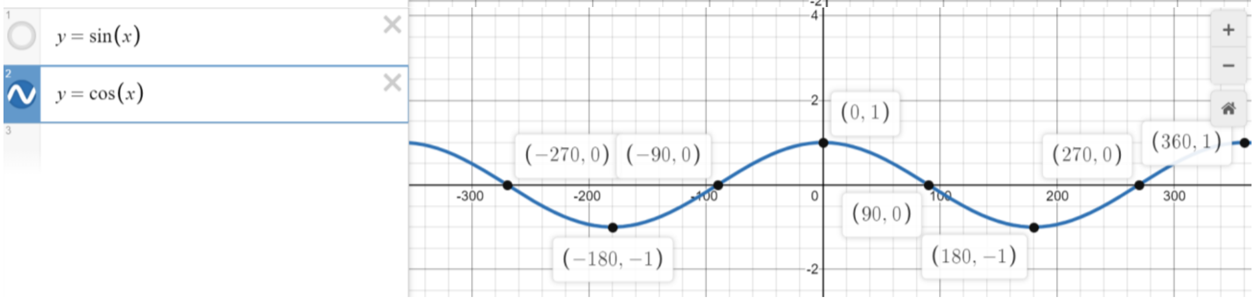
$$\sin(0) = 0, \sin(90) = 1, \sin(180) = 0, \sin(270) = -1, \sin(360) = 0$$
$$\cos(0) = 1, \cos(90) = 0, \cos(180) = -1, \cos(270) = 0, \cos(360) = 1$$

“All Students Take Calculus”

All	Students	Take	Calculus
I	II	III	IV
All the trig functions are positive	Sine function is positive	Only tangent is positive	Only cosine is positive
$\sin(x) > 0$	$\sin(x) > 0$	$\sin(x) < 0$	$\sin(x) < 0$
$\cos(x) > 0$	$\cos(x) < 0$	$\cos(x) < 0$	$\cos(x) > 0$
$\tan(x) > 0$	$\tan(x) < 0$	$\tan(x) > 0$	$\tan(x) < 0$

When put values of sine and cosine on a traditional x-y planar graph (x on the horizontal and sine/cosine value on the vertical axis), we get a graph type called sinusoidal.

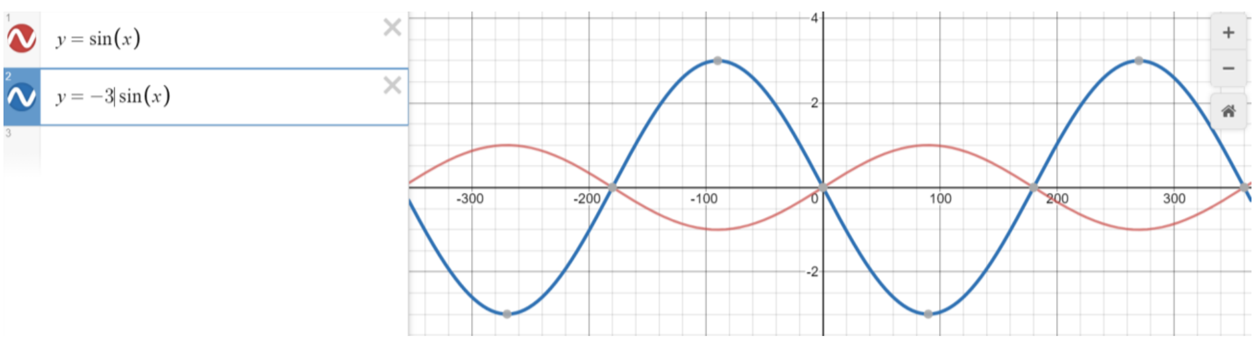


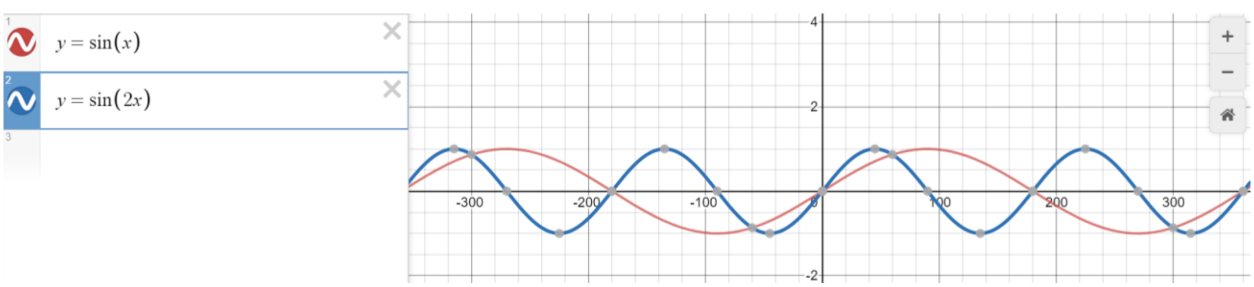


Amplitude: the distance from the midline of function (0) to the maximum height of the function (1)
Wavelength : the distance between peaks of the graph (360-degrees)
Frequency: is how quickly the graph goes through one cycle
Period: time it takes to go through one cycle
Phase shift: horizontal (right/left) shift in the graph

$$y = Asin(Bx + C)$$

A is the Amplitude

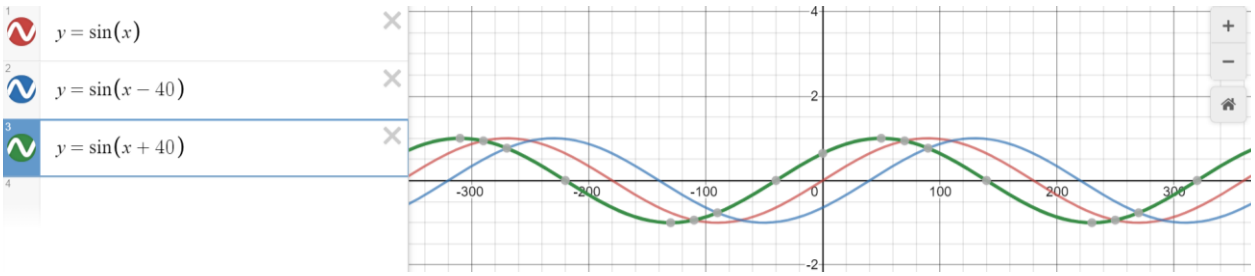




Wavelength = $\frac{360}{B}$

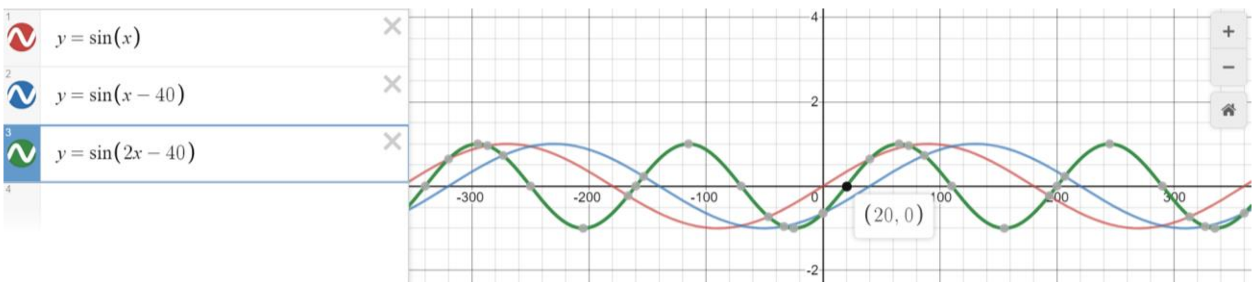
Period ~ Wavelength if you are measuring the wavelength in time units

(angular) Frequency is the reciprocal of the period $F = \frac{1}{P} = \frac{B}{360}$



$$y = Asin(Bx + C)$$

Phase shift is $-\frac{C}{B}$

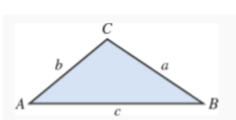


$$y = 6\cos(9x - 180)$$

Amplitude = 6
Period = $\frac{360}{9} = 40$
Phase shift = $-\frac{-180}{9} = 20$

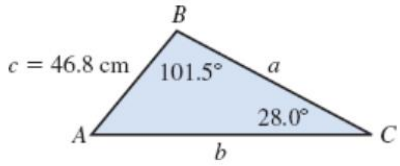
14.1/14.2 mostly about understanding how we get wavy graphs, and the idea of extending the functions beyond the values for acute angles

The rest of the chapter (14.3/14.4) are on obtuse/acute triangles (not right triangles)
Law of Sines
Law of Cosines



Law of Sines: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



2 angles and a side : SAA
One angle must be opposite of the given sides.

$$\frac{\sin(C)}{c} = \frac{\sin(28)}{46.8} = \frac{\sin(B)}{b} = \frac{\sin(101.5)}{b}$$

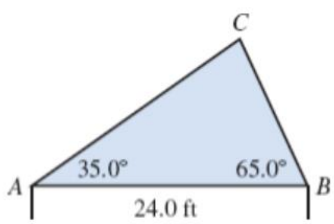
$$\frac{\sin(28)}{46.8} = \frac{\sin(101.5)}{b}$$

$$b(\sin(28)) = 46.8\sin\,(101.5)$$
$$b = \frac{46.8\,\sin(101.5)}{\sin(28)}$$
$$b = 97.7$$

Missing angle A = 180-101.5-28=50.5

$$\frac{\sin(C)}{c} = \frac{\sin(28)}{46.8} = \frac{\sin(A)}{a} = \frac{\sin(50.5)}{a}$$

$$a = \frac{46.8\,\sin(101.5)}{\sin(50.5)} = 59.43$$



ASA ~ find the missing angle opposite the given side.

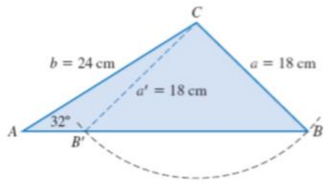
180-65-35=80=C

$$\frac{\sin(80)}{24} = \frac{\sin(35)}{a}$$

$$\frac{\sin(80)}{24} = \frac{\sin(65)}{b}$$

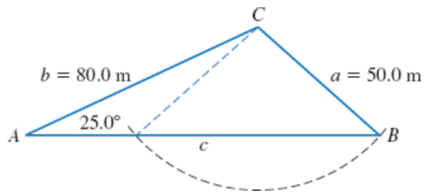
Solve for a and b

Two possible triangles can be drawn—one with an obtuse angle and two acute angles and one with all acute angles.



Typically, if the angle you are given is small, and two sides: may have no triangle, or you may have two different triangles.
SSA

Sine function has the same value for two different angles.



$$A = 25°, a = 50.0, b = 80.0$$

$$\frac{\sin(A)}{a} = \frac{\sin(25)}{50.0} = \frac{\sin(B)}{80}$$
$$\frac{80\sin(25)}{50} = \sin(B) = 0.676189\ldots$$

$$\sin^{-1} 0.676189\ldots = 42.5$$

The angle that has he same sine value as 42.5 is 180-42.5 = 137.5

25+137.5=162.5 this is less than 180, then we can find a third angle that will make another triangle.

Triangle 1: 25, 42.5, 112.5

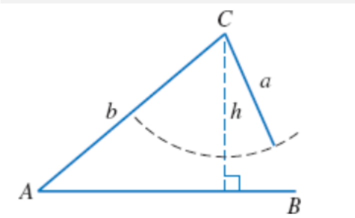
$$\frac{\sin(112.5)}{c} = \frac{\sin(25)}{50.0}$$

$$c = \frac{50\sin(112.5)}{\sin(25)} = 109.3$$

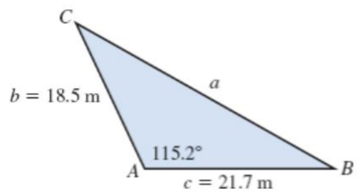
Triangle 2: 25, 137.5, 17.5

$$\frac{\sin(17.5)}{c} = \frac{\sin(25)}{50.0}$$

$$c = \frac{50\sin(17.5)}{\sin(25)} = 35.6$$



Law of Cosines



SAS ~ angle is opposite the missing side (in between the given sides)
SSS ~ all three sides and no angles

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

$$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab} = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$a^2 = (18.5)^2 + (21.7)^2 - 2(18.5)(21.7)\cos(115.2)$$
$$a^2 = 1154.998..$$
$$a = 33.98 \approx 34$$

Once you have the missing side, you can switch back to the law of sines.
There are never two triangles.

$$\frac{\sin(115.2)}{34} = \frac{\sin(B)}{18.5}$$

Third angle: subtract the two you found from 180.