

10/09/2020

Chapter 8

Desmos: <https://www.desmos.com/calculator>

Linear Equations: two-variable equations that make a line

Linear: variables that have no powers, x , y , etc. including $3x$, $-5y$, etc.
Equations that have two variables:

Standard form of a line in 2D is $Ax + By = C$

Will draw a straight line even if the coefficient of one of the variables is 0.

[General form: $Ax + By + C = 0$]

A solution to a linear equation in 2 variables is going to come in a pair of values (x, y) (an ordered pair)
Always write the variables in the same order, with x first and y second

There is an infinite number of solutions.

Select one the variables, and then put into the equation to find the value of the other variable that goes with it.

$$3x + 2y = 12$$

$x = 1$ what does that mean y has to be?

$$3(1) + 2y = 12$$

$$3 + 2y = 12$$

$$2y = 9$$

$$y = \frac{9}{2} = 4.5$$

$$(1, 4.5) = \left(1, \frac{9}{2}\right)$$

$x = 0$ then y has to be a different value.

$$3(0) + 2y = 12$$

$$2y = 12$$

$$y = 6$$

$$(0, 6)$$

Likewise $y = 3$ and that will force x to be something in particular

$$3x + 2(3) = 12$$

$$3x + 6 = 12$$

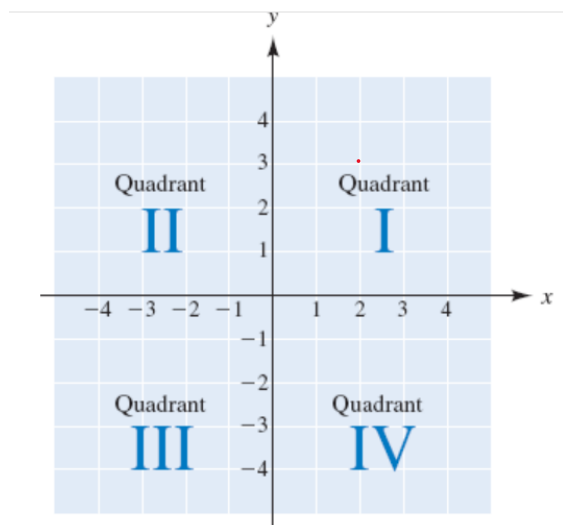
$$3x = 6$$
$$x = 2$$

(2,3)

Always write points in the order x first, then y second

We can't list all the possible solutions, but we can draw a graph that represents all the solutions. Plot the points, the ordered pairs. Connect the dots.

How do we plot a coordinate pair of points?



x values are plotted on the horizontal axis, and y values are plotted on the vertical axis

(2,3)... going to move 2 units in the x -direction (2 units right), and then 3 units in the y -direction (3 units up).

(-2,3)... move 2 to the left

(-2,-3)... move 2 left and down 3

(2,-3) ... move down 3

Quadrant I : always contains points that have both coordinates positive.

Quadrant II: have x -coordinate negative, but y -coordinates positive

Quadrant III: have coordinates negative

Quadrant IV: x -coordinate positive, y -coordinate negative

If one of the coordinates is 0, then the point lies on one of the axes.

(0,4) nowhere left-right, but up 4, on y -axis

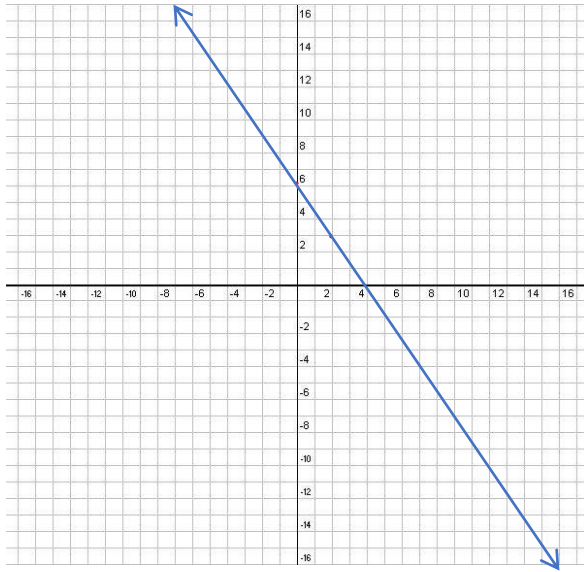
(4,0), right 4, but not up or down at all, on x-axis
Where the axes cross is (0,0), called the origin

Our line: $3x + 2y = 12$

Coordinates that were on the line:

(0,6), (2,3)

To draw a line, we need two points.



Any pair of points that falls on your plotted line is a solution to the equation.

(4,0) test this in our equation

$$3(4) + 2(0) = 12$$

(10,-9)

$$3(10) + 2(-9) = 30 - 18 = 12$$

Eyeballing is fraught!

Try to label the points on the graph that you used to plot it.

Intercepts are where the graph crosses one axis (one coordinate is zero)

X-intercept is where the graph crosses the x-axis

And the Y-intercept is where the graph crosses the y-axis

$$3x + 2y = 12$$

Solve for y:

$$y = mx + b$$

Slope-intercept form (it's the version of the line where you've solved for y)

$$m = \text{slope}$$

$$b = y - \text{intercept}$$

$$2y = -3x + 12$$

$$y = -\frac{3}{2}x + 6$$

Plotting lines by using intercepts:

- 1) Solve for the intercepts: replace one variable with 0 and find the other, and then switch.
- 2) Plot points and draw the line between them.

Special cases: vertical and horizontal lines.

Both variables: slanted lines

Equations that contain only y produce horizontal lines $y = 4$

Equations that contain only x produce vertical lines $x = -2$

Only has one intercept

$$y = x$$

Only has one intercept, but only because they are both (0,0)

If you are asked to plot an equation like this, choose a second point of your choosing.

Slope

Is a way of measuring how rapidly y is increasing or decreasing as x increases

Grade of a road = slope

5% grade it means that for every 100 feet you increase in x, the y height increases by 5 feet

Slope of a roof (ratio)

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope from two pairs of points.

$$(0,6), (2,3)$$

$$(x_1, y_1), (x_2, y_2)$$

$$\text{slope} = m = \frac{3 - 6}{2 - 0} = -\frac{3}{2}$$

For every 2 units in x (positive), the y-coordinate decreases by 3

Slope-intercept form of the line gives you the slope.

$$2x + 5y = 10$$

Option 1: to find two points on the line and find the slope from the points.

Option 2: is to solve for y, and read the slope from coefficient of x

How to plot a graph using a point and a slope.

$$(2,2), m = \frac{4}{3}$$

(remember that a whole number can be written as a fraction $2 = \frac{2}{1}$)

Start at (2,2)

Slope says that x increases by 3 : $x + 3 = 2 + 3 = 5$

And then y increases by 4: $y + 4 = 2 + 4 = 6$

New coordinate is (5,6)

Now I have two points from which I can draw a line.

(Later, we'll be able to write an equation to test also)

Parallel and Perpendicular lines

Parallel line is that they never intersect

In algebra, this means that the lines have to have the same slope (with different intercept)

$$y = \frac{2}{3}x + 5$$

$$y = \frac{2}{3}x + 1$$

Perpendicular lines intersect at a 90-degree angle

Horizontal and vertical lines are always perpendicular

The slopes of perpendicular lines are negative reciprocals (flip the fraction and change the sign)

$$m_1 \cdot m_2 = -1$$

$$y = \frac{2}{3}x + 5$$

$$y = -\frac{3}{2}x + 1$$

Slope of a horizontal line is 0:

$$m = \frac{(0,4), (3,4)}{4 - 4} = \frac{0}{3 - 0} = 0$$

Slope of a vertical line:

$$\frac{(2,1), (2,5)}{5 - 1} = \frac{4}{2 - 2} = \frac{4}{0} = \text{undefined}$$

It does kinda fit, but it may be helpful to just think of this as a special case.

$$3x + 4y = 12$$
$$3x + 4y = 28$$

Vs.

$$5x - 2y = 10$$
$$2x + 5y = 10$$

Solve the equation for y first, and then compare the slopes.

Write an equation of a line.

Point-Slope Version of a Line

$$y - y_1 = m(x - x_1)$$

(x_1, y_1) is the point, m is the slope that you are given.

$$(2, 2), m = \frac{4}{3}$$

$$y - 2 = \frac{4}{3}(x - 2)$$

$$y - 2 = \frac{4}{3}x - \frac{8}{3}$$

$$y = \frac{4}{3}x - \frac{2}{3}$$

Slope-intercept form.

$$6 = ? \frac{4}{3}(5) - \frac{2}{3} = \frac{20}{3} - \frac{2}{3} = \frac{18}{3} = 6$$

Slope-intercept can be used to be find the same equation.

$$y = mx + b$$

$$2 = \left(\frac{4}{3}\right)(2) + b$$

$$2 = \frac{8}{3} + b$$

$$b = -\frac{2}{3}$$

$$y = \frac{4}{3}x - \frac{2}{3}$$

Most difficult kind of problem we'll encounter is they will ask for the equation of the line given only two points.

$$(4,3)(-5,1)$$

- 1) Find the slope
- 2) Use the slope and 1 point to find the equation.

$$m = \frac{1 - 3}{-5 - 4} = -\frac{2}{-9} = \frac{2}{9}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{2}{9}(x - 4)$$

$$y - 3 = \frac{2}{9}x - \frac{8}{9}$$

$$y = \frac{2}{9}x + \frac{19}{9}$$

Slopes that are positive go up to the right

Slopes that are negative go down to the right

Slopes that are zero are horizontal

Slopes that are undefined go up and down

$y = x$ has a slope of 1.

Slope is a fraction (less than 1) the line should fall below the line $y = x$ (closer to the x-axis)

Slope is bigger than 1, the line should fall above the line $y = x$ (closer to the y-axis)