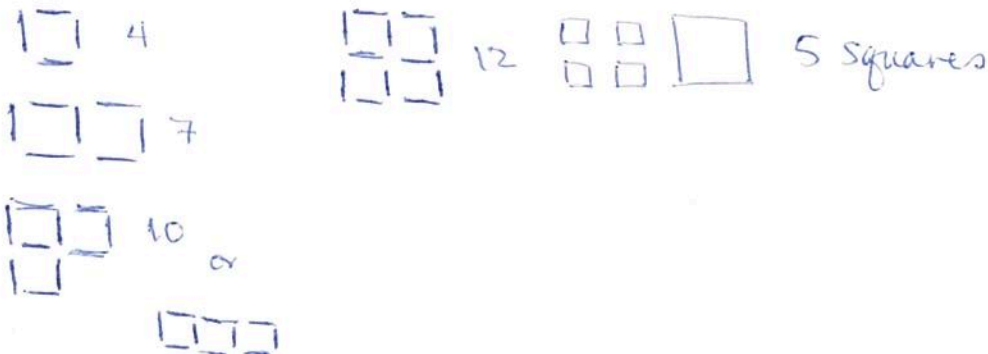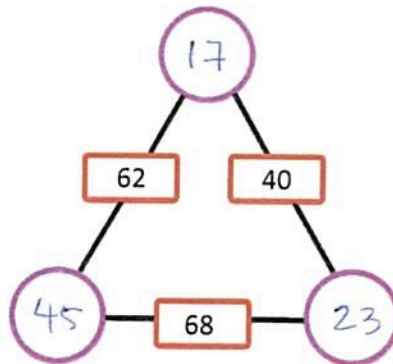**Instructions**: Show all work. Partial credit can only be given where work is shown. Be sure to answer all parts of each question. You may not use a calculator on this quiz.

1. You can make one square with four toothpicks. Show how you can make two squares with seven toothpicks (breaking toothpicks is not allowed), three squares with 10 toothpicks, and five squares with 12 toothpicks. (10 points)
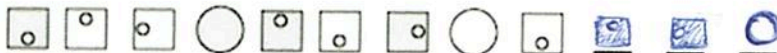

2. Solve the arithmagon. What numbers must go in the vertices of the triangle so that the two numbers at the end of each side adds to the number shown on the side? (9 points)

$$x+y=40 \rightarrow y=40-x$$
$$y+z=68 \rightarrow 40-x+z=68$$
$$x+z=62 \qquad -x+z=28$$
$$\underline{-x+z=28}$$
$$2z=90 \qquad x+45=62$$
$$z=45 \qquad x=17$$
$$y=40-17=23$$

$17+45=62$
$17+23=40$
$45+23=68$ ✓

3. Give the next three terms in the sequence. (9 points)

4. When Mickey counted the number of outcomes of rolling two 4-sided tetrahedral dice and got (1,1), (2,2), (3,3), (4,4), (1,2), (3,4), (2,4), (4,1). He started to get confused about which ones he had listed and which ones were left. How would you help him create a more systematic list? Complete the list. (10 points)

build a table
* already in list

	1	2	3	4	← die #1
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

↑
die #2

still needs
(1,3), (1,4), (2,1), (2,3), (3,1), (3,2),
(4,2), (4,3).

5. Write the number 2953 in
   a) Roman numerals (5 points)

   MMCMLIII

   b) Babylonian (6 points)

   <<<< ▽▽▽ < ▽▽▽

6. Translate each number from the historical writing system back into the Hindu-Arabic system.

   a. (7 points)   7×1 + 1(500) + 5(400) + 6(8000) = 50,307

   b. (6 points)
   6000 + 400 + 10+3   6413

   c. MMMCDXCVII (5 points)

   3497

7. Write the decimal number 531 as a number in base-seven. (8 points)

$$1 \times 7^3 + 3 \times 49 + 5 \times 7 + 6 \times 1$$

$$1356 \text{ seven}$$

$7^0 = 1, 7^1 = 7, 7^2 = 49$
$7^3 = 343$

$531 - 343 = 188$
$188 - 147 = 41$
$41 - 35 = 6$

8. Is the statement $\{4,3\} \subset \{1,3,4,6\}$ true or false? (3 points—no partial credit)

True

9. Consider the sets $A = \{a, h, k, l, p, v, y\}, B = \{b, g, j, k, m, p, u, x\}$. Find the following if the Universal set is the set of all (lower case) letters in the Alphabet. (4 points each)

   a. $A \cup B$
   $$\{a, b, g, h, j, k, l, m, p, u, v, x, y\}$$
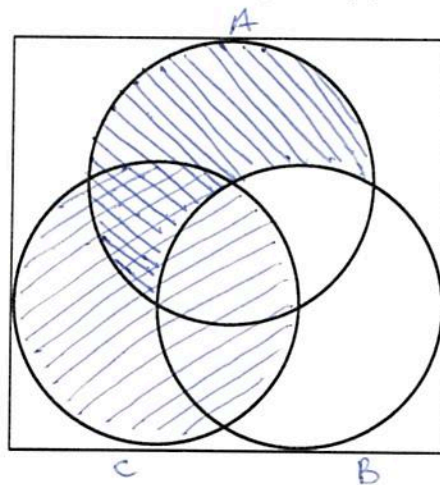
   b. $A \cap B$
   $$\{k, p\}$$

   c. $B - A$
   $$\{b, g, j, m, u, x\}$$

   d. $\bar{A}$
   $$\{b, c, d, e, f, g, i, m, n, o, q, r, s, t, u, w, x, z\}$$

10. On the Venn diagram below, shade the region given by $(A - B) \cup C$. Label your sets clearly. (8 points)

all shading in

$(A-B) \cup C$

11. Use the distributive property (with either addition or subtraction to find the value of 33(18) by hand. Show all work. You may check your answer in a calculator, but a correct answer alone will not be worth full credit. (7 points)

$$33(20-2) = 33(20) - 33(2) = 660 - 66 = 594$$

12. Verify the result above by using the Russian Peasants' method of multiplication. (6 points)

~~18 × 33~~
9 × 66
~~4 × 132~~
~~2 × 264~~
1 × 528

$$\overset{1}{5}28$$
$$\underline{66}$$
$$594$$

13. Complete the base-four multiplication table, and then use it to find the value of $203_{four} \div 11_{four}$. (10 points) $= 13_{four}$

×	1	2	3	10	11	12	13
1	1	2	3	10	11	12	13
2	2	10	12	20	22	30	32
3	3	12	21	30	33	102	111
10	10	20	30	100	110	120	130
11	11	22	33	110	121	132	203
12	12	30	42	102	132	210	222
13	13	32	111	130	203	222	301

14. Compute the dividend and remainder of 8)$\overline{103}$ using repeated subtraction. (7 points)

103 − 8 = 95
95 − 8 = 87
87 − 8 = 79
79 − 8 = 71
71 − 8 = 63
63 − 8 = 55

55 − 8 = 47
47 − 8 = 39
39 − 8 = 31
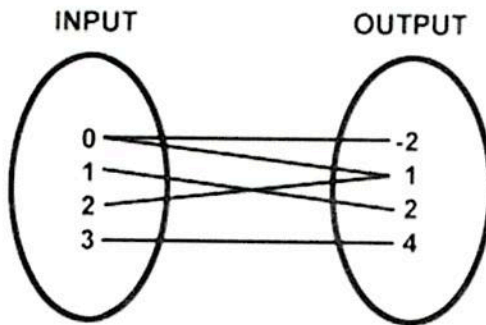31 − 8 = 23
23 − 8 = 15
15 − 8 = 7

12 R 7

15. Simplify the expression $162 - 2 \times 3^2(27 + 115)^0$ using order of operations. (7 points)

$$162 - 2 \times 9(1) = 162 - 2 \times 9 =$$
$$162 - 18 = 144$$

16. For the relation shown, write out the list of ordered pairs that represents the relation. Then determine if the relation is a function. (10 points)

INPUT   OUTPUT

$\{(0, -2), (0, 1), (1, 2), (2, 1), (3, 4)\}$

it is not a function

Bonus: Write the hexadecimal number $A81FB_{sixteen}$ as a base-five number. (15 points)

$$\begin{array}{ccccc} A \cdot & 8 & 1 & F & B \end{array}$$
$$10 \times 16^4 + 8 \times 16^3 + 1 \times 16^2 + 15(16) + 11(1)$$
$$655,360 + 32768 + 256 + 240 + 11$$
$$= 688,635. \ \text{base-10}$$

$5^0 = 1, \ 5^1 = 5, \ 5^2 = 25, \ 5^3 = 125, \ 5^4 = 625, \ 5^5 = 3125, \ 5^6 = 15,625, \ 5^7 = 78,125$
$5^8 = 390,625$

$$1 \times 5^8 + 3 \times 5^7 + 4 \times 5^6 + 0 \times 5^5 + 1 \times 5^4 + 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 0 \times 5^0$$

$$\boxed{134014020_{five}}$$

$688,635 - 390,625 = 298,010$ $63635 - 62,500 = 1135$ $510 - 500 = 10$
$298,010 - 234,375 = 63635$ $1135 - 625 = 510$ $10 - 10 = 0$