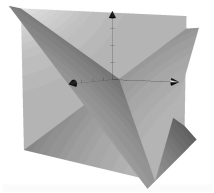
# Intersection of Three Planes

Consider the following system of three equations, where the third equation is formed by taking the sum of the first two. Geometrically, each equation can be thought of as a plane in $\mathbb{R}^3$.

$$\begin{cases} x + y - 2z &= 3 \\ x - y + z &= 2 \\ 2x - z &= 5 \end{cases}$$

1. Without doing any calculations, what do you think the intersection of these three planes looks like? Choose the answer below that most closely aligns with your thinking, and explain your reasoning.

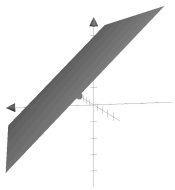
   a. The intersection is some point in $\mathbb{R}^3$.

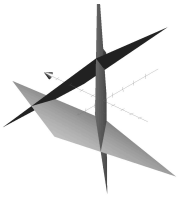
   b. The intersection is some line in $\mathbb{R}^3$.

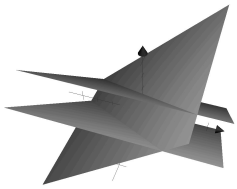
   c. The intersection is some plane in $\mathbb{R}^3$.

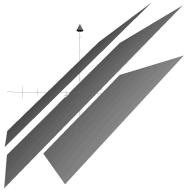
   d. The three planes have no common point(s) of intersection, but each pair of planes intersect in a line in $\mathbb{R}^3$.

   e. The three planes have no common point(s) of intersection, but one plane intersects each plane in a pair of parallel planes.

   f. The three planes have no common point(s) of intersection; they are parallel in $\mathbb{R}^3$.

   g. None of the above.

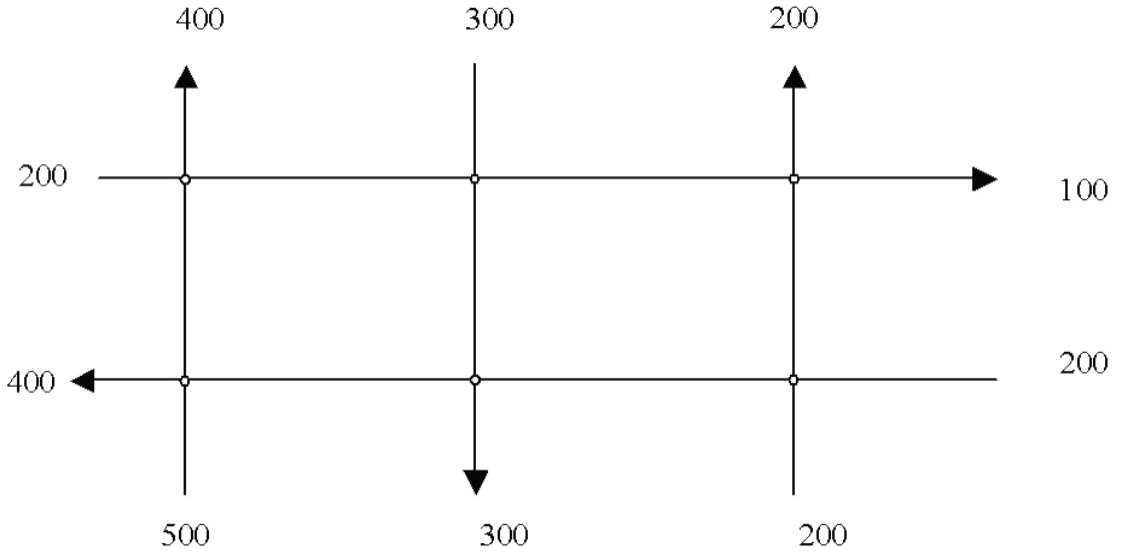
   h. There is no way to know unless we do some calculations.

2. Using any method you like, determine an algebraic representation of the intersection of the three planes that supports your choice given in #1.

3. In what ways, if any, does the intersection of the three planes in #1 relate to the existence and uniqueness of solution(s) to the system of equations in #1? Explain your reasoning.

# The Traffic Flow Task

The following diagram represents the street plan of the busiest two blocks in the financial district of a city. The Traffic Control Center has installed electronic sensors that count the number of vehicles passing through specific points in the city. The arrows represent the direction of each street and the numbers tell how many vehicles the electronic sensors count as passing that way each hour. At each intersection, a roundabout directs traffic and allows for a continuous flow of traffic through the entire system. Cars are not allowed to park on the streets.




*Street Plan and Flow of Traffic*

The Traffic Control Center is interested in analyzing possible traffic diversion policies, which are needed when traffic disruptions occur (such as construction, parades, or presidential inaugurations). Your job is to assist the Traffic Control Center with this analysis. In your analysis, assume that the traffic flow must follow its usual course at the sensor locations.

1. **Minimize Traffic Flow:** If you were able to set minimum quantities of cars to circulate in a road segment between roundabouts, what would this amount be for each segment to maintain the traffic flow values given at the sensor locations on the map? Note that cars may be diverted from their desired entry and exit for these two blocks as long as the overall flow at the sensors remains as stated.

2. **Signage:** As part of determining these minimums, which segments of road may be totally closed, i.e., no traffic flow at all on these segments? Can more than one segment be closed at a time? If so, which pairs or triplets of segments may be closed?

3. **Special Scenario:** Is there a scenario that would allow for no more than 200 cars on each of the seven internal segments?

Your group should begin by following the traffic through the diagram to determine possible scenarios for road closings and what restrictions those closings would place on other road segments. Be prepared to give some initial predictions of how many and which roads may be closed based on your investigation of the diagram. After getting some initial predictions, create an algebraic description of the situation that can help you answer these questions.

This is a modification of a problem developed by researchers in Depto de Matemáticas ITAM in Mexico City, Mexico. Used with permission of Professor María Trigueros Gaisman.

# Example Generation: Equations, Unknowns, and Intersections

Write a system of linear equations and the row reduced echelon form (RREF) of the corresponding augmented matrix that meets the requirements described in the table. If no such system exists, state this and explain why.

| | No intersection | Intersects in a point | Intersects in a line | Intersects in a plane |
|---|---|---|---|---|
| 2 equations & 2 unknowns | | | | |
| 2 equations & 3 unknowns | | | | |
| 3 equations & 2 unknowns | | | | |
| 3 equations & 3 unknowns | | | | |

Write at least 2 generalizations that can be made from these examples and the strategies you used to create them.