MTH 266, IOLA Homework, Unit 1   Name ____

Instructions: Answer the following questions and attach your answers to this page for submission.

1. Consider the expression: $4\begin{bmatrix}1\\-3\end{bmatrix}-2\begin{bmatrix}3\\5\end{bmatrix}$,
   a. Sketch a graphical representation of this expression using vectors.
   b. Describe in words what the scalars 4 and -2 represent in the sketch.

2. In class, we decided that it was impossible to reach Old Man Gauss with only one of the modes of transportation (hover board or magic carpet). Write 2 different convincing arguments that justify this conclusion. You may make use of sentences, calculations, sketches, etc. – anything that helps support your justification.

3. After some discussion, we came to an agreement in class that (allowing for negatives and decimal increments), everywhere on the plane could be reached by using the hover board $\begin{bmatrix}3\\1\end{bmatrix}$ and the magic carpet $\begin{bmatrix}1\\2\end{bmatrix}$ (i.e., there was nowhere that Gauss could hide). We learned that another way to say this is that the $span\left\{\begin{bmatrix}3\\1\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix}\right\}=\mathbb{R}^2$.
   a. Make a geometric argument supporting this claim. Include a graphical illustration as well as a verbal explanation.
   b. Make an algebraic argument supporting this claim using vector notation and linear combinations of vectors.

4. Now suppose your modes of transportation are given by $\begin{bmatrix}a\\b\end{bmatrix}$ and $\begin{bmatrix}c\\d\end{bmatrix}$, where a, b, c, d are real numbers. What must be true about the relationship(s) among a, b, c, and d in order to ***still*** be able to reach any point in $\mathbb{R}$!? Justify your reasoning.

5. Suppose $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are vectors in $\mathbb{R}^2$. Is $\begin{bmatrix}0\\0\end{bmatrix}$ in *span* $\{\boldsymbol{v_1}, \boldsymbol{v_2}\}$? Explain your reasoning in enough detail to justify your conclusion.

6. Name one vector in $\mathbb{R}^3$ that is in the span of $\left\{\begin{bmatrix}2\\3\\1\end{bmatrix},\begin{bmatrix}-4\\-6\\-2\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$ and one vector in $\mathbb{R}^3$ that is not in the span of $\left\{\begin{bmatrix}2\\3\\1\end{bmatrix},\begin{bmatrix}-4\\-6\\-2\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$. Explain your reasoning.

7. Suppose we have ***three*** different modes of transportation that allow us to move in the plane and we are still trying to find Gauss's cabin in that plane. In other words, we are considering a set of three vectors in $\mathbb{R}^2$ and whether the span of those three vectors is all of $\mathbb{R}^2$ or not.

   For each part below, state the vectors requested (in which case no further explanation is necessary to receive credit) or explain carefully why it is not possible to provide such vectors.

   a. List a set of three vectors in $\mathbb{R}^2$ that does not span $\mathbb{R}^2$.
   b. List a set of three vectors in $\mathbb{R}^2$ that spans $\mathbb{R}^2$.
   c. List a set of three vectors in $\mathbb{R}^2$ that spans $\mathbb{R}^2$ from which you can remove one vector and still span $\mathbb{R}^2$ with the remaining two vectors. State which vector can be removed.

d. List a set of three vectors in $\mathbb{R}^2$ that spans $\mathbb{R}^2$ from which you can remove two vectors and still span $\mathbb{R}^2$ with the remaining one vector. State which vectors can be removed.
e. List a set of three vectors in $\mathbb{R}^2$ that spans $\mathbb{R}^2$ from which you cannot remove any one of the three vectors and have the remaining two vectors span $\mathbb{R}^2$.

8. Suppose $S = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\} \subset \mathbb{R}^3$ such that $span\ S = \mathbb{R}^3$. What, if anything, can be said about solution(s) to the equation $c_1\boldsymbol{v_1} + c_2\boldsymbol{v_2} + c_3\boldsymbol{v_3} = \boldsymbol{0}$? (assume the c_i are real numbers). [Please do not look to resources to help you answer this. I want to learn about your initial ideas on this problem. If you are not sure about what conclusions can/can't be made, then articulate that. A thoughtful response, regardless of "correctness," will get full credit.]

9. Suppose you have three modes of transportation. The broomstick's movement is by the vector $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$. By this we mean that if the broomstick traveled forward for one hour, it would move along a "diagonal" path that would result in a displacement of 1 unit West and 2 units North of its starting location. A new magic carpet's movement is defined by the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and the jet pack's movement by the vector $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$.
   a. Write a vector equation to describe traveling to the location 3 East and 6 South of the starting location, i.e., $\begin{bmatrix} 3 \\ -6 \end{bmatrix}$. Explain all parts of your equation in terms of the travel situation.
   b. Write a system of equations that has the same solution as the vector equation you wrote in 9(a). Can $\begin{bmatrix} 3 \\ -6 \end{bmatrix}$ be reached using these three modes of transportation? (circle one).
      i. Yes, there is exactly one way to do this.
      ii. Yes, there are many ways to do this.
      iii. No, there is no way to do this.

   Explain why you circled your choice. If possible, give an example of a way to make this trip using the three modes of transportation by listing the amount of time travelled on each. Otherwise, make clear why it is not possible to do so.

10. Recall the three modes of transportation in the 3D world: $\boldsymbol{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \boldsymbol{v_2} = \begin{bmatrix} 6 \\ 3 \\ 8 \end{bmatrix}, \boldsymbol{v_3} = \begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix}$.
   a. Is there anywhere in the 3D world that Gauss could hide from you? If so, name a location and say why you know he could hide there. If there is nowhere he can hide, why not?
   b. (If you answered he could hide in part a): how could you change the modes of transportation so that he could not hide from you? Give an example.

11. Suppose $\boldsymbol{v_1} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \boldsymbol{v_2} = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, \boldsymbol{v_3} = \begin{bmatrix} 5 \\ 7 \\ h \end{bmatrix}$.

   a. For what value(s) of h is v_3 in $span\ \{\boldsymbol{v_1}, \boldsymbol{v_2}\}$? How do you know?
   b. For what value(s) of h is $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ linearly dependent? How do you know?

12. True or False: If $\boldsymbol{x}$, $\boldsymbol{y}$ is linearly independent but $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$ is linearly dependent, then $\boldsymbol{z}$ is in $span\ \boldsymbol{x}, \boldsymbol{y}$. If true, explain why. If false, provide a counterexample.

13. Recall the six generalizations we created in class:
    a. If a set of vectors contains exactly 2 vectors and those vectors are scalar multiples of each other, then the set is linearly dependent.
    b. If a set of vectors contains at least 2 vectors that are scalar multiples of each other, then the set is linearly dependent.
    c. If a set contains p vectors in $\mathbb{R}$ and $p > n$, then the set is linearly dependent.
    d. If a set of vectors contains the zero vector, then the set is linearly dependent.
    e. If a set contains exactly one vector and it is nonzero, then the set is linearly independent.
    f. A set of vectors in linearly dependent if and only if at least one vector in the set can be written as a linear combination of other vectors in the set.

    Choose two of these generalizations and for each, provide an explanation that justifies them as true. These explanations are like "mini-proofs," so they should contain full sentences that help the reader following your mathematics. You can also use sketches, logic, computations, etc. – whatever helps!

14. Complete the worksheet from class that asked you for examples of linearly independent and dependent sets. No explanation. For each example that you created, though, also describe the span of that set (e.g., you could say the span is the set of all vectors that are multiples of $\begin{bmatrix}3\\5\end{bmatrix}$, which is a line in $\mathbb{R}$!). The handout is replicated on the back, with the span part added in.

	Linearly dependent set	Linearly independent set
A set of 2 vectors in R^2		
Span of the set		
A set of 3 vectors in R^2		
Span of the set		
A set of 2 vectors in R^3		
Span of the set		
A set of 3 vectors in R^3		
Span of the set		
A set of 4 vectors in R^3		
Span of the set		