Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Solve the homogeneous differential equation $xy' = y(\ln x - \ln y)$.

$y = vx$

$y' = \frac{y}{x}\left(\ln\left(\frac{x}{y}\right)\right)$ $y' = v'x + v$

$v'x + v = \frac{vx}{x}\left(\ln\left(\frac{x}{vx}\right)\right)$ $\frac{y}{x} = v$

$v'x + v = v\ln\left(\frac{1}{v}\right) = -v\ln v$

$v'x = -v\ln v - v$

$\int \frac{dv}{v(\ln v + 1)} = \int -\frac{1}{x}dx$

$\ln(\ln v + 1) = -\ln x + C = \ln\left(\frac{A}{x}\right)$

$\ln v + 1 = \frac{A}{x}$

$u = \ln v + 1$

$du = \frac{1}{v}dv$

$\ln v = \frac{A}{x} - 1$

$v = e^{\frac{A}{x}-1}$

$\frac{y}{x} = e^{\frac{A}{x}-1}$

$y = xe^{\frac{A}{x}-1}$

2. Solve the Bernoulli equation $y' - \frac{2}{x}y = \frac{x}{y^2}$.

$y' - \frac{2}{x}y = xy^{-2}$ $n = -2$ $(1-n)y^{-n} = 3y^2$

$3y^2y' - \frac{6}{x}y^3 = 3x$

$z' - \frac{6}{x}z = 3x$ $z = y^3$

$z' = 3y^2y'$

$\mu = e^{\int -\frac{6}{x}dx} = e^{-6\ln x} = x^{-6}$

$x^{-6}z' - 6x^{-7}z = 3x^{-5}$

$\int (x^{-6}z)' = \int 3x^{-5}dx$

$x^{-6}z = -\frac{3}{4}x^{-4} + C \implies z = -\frac{3}{4}x^2 + Cx^6$

$\implies y^3 = -\frac{3}{4}x^2 + Cx^6$

3. Classify the differential equation by order, linearity and whether it is ordinary or partial.

   a. $\frac{d^2y}{dx^2} = x\cos y$

   Nonlinear, ordinary
   2nd order

   b. $\left(\frac{\partial u}{\partial x}\right)^3 - \frac{\partial^2 u}{\partial x\partial y} = uxy$

   nonlinear, partial, 2nd order