MTH 267, Quiz #3, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Solve the differential equation $\frac{dy}{dx} = 2xy^2 + x$ using separation of variable for the general solution.

$\frac{dy}{dx} = x(2y^2+1) \Rightarrow \frac{dy}{2y^2+1} = x\,dx \Rightarrow \frac{1}{\sqrt{2}}\arctan(\sqrt{2}y) = \frac{1}{2}x^2 + C$

$\Rightarrow \arctan(\sqrt{2}y) = \frac{\sqrt{2}}{2}x^2 + C$

$\Rightarrow \sqrt{2}y = \tan\left(\frac{\sqrt{2}}{2}x^2 + C\right)$

$y = \frac{1}{\sqrt{2}}\tan\left(\frac{\sqrt{2}}{2}x^2 + C\right)$

2. Solve the differential equation $4y' + 12x^3y = x^3$, $y(1) = 1$ using the method of integrating factors (reverse product rule) for the particular solution.

$y' + 3x^3y = \frac{1}{4}x^3$ $\mu = e^{\int 3x^3dx} = e^{\frac{3}{4}x^4}$

$e^{\frac{3}{4}x^4}y' + 3x^3e^{\frac{3}{4}x^4}y = \left(\frac{1}{4}x^3\right)e^{\frac{3}{4}x^4}$

$\int\left(e^{\frac{3}{4}x^4}y\right)' = \int\frac{1}{4}(x^3)e^{\frac{3}{4}x^4}$

$y = e^{-\frac{3}{4}x^4}\int\frac{1}{4}(x^3)e^{\frac{3}{4}x^4}\,dx$

$= e^{-\frac{3}{4}x^4}\left[\frac{1}{12}e^{\frac{3}{4}x^4} + C\right]$

$= \frac{1}{12} + Ce^{-3/4x^4}$