MTH 267, Quiz #12, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the particular solution to the second order equation $y'' - 5y' - 14y = 4e^{2t} - 9\cos 3t$ using the method of undetermined coefficients.

$r^2 - 5r - 14 = 0$

$(r-7)(r+2) = 0$

$r = 7, r = -2$

$y_h = c_1 e^{7t} + c_2 e^{-2t}$

$Y_{p_1} = Ae^{2t}$

$Y_{p_1}' = 2Ae^{2t}$

$Y_{p_1}'' = 4Ae^{2t}$

$4Ae^{2t} - 10Ae^{2t} - 14Ae^{2t} = 4e^{2t}$

$-20A = 4$

$A = -\frac{1}{5}$

$Y_{p_1} = -\frac{1}{5}e^{2t}$

$Y_{p_2} = A\cos 3t + B\sin 3t$

$Y_{p_2}' = -3A\sin 3t + 3B\cos 3t$

$Y_{p_2}'' = -9A\cos 3t - 9B\sin 3t$

$-9A\cos 3t - 9B\sin 3t + 15A\sin 3t - 15B\cos 3t - 14A\cos 3t - 14B\sin 3t = -9\cos 3t$

$\cos 3t(-9A - 15B - 14A) = -9$

$\sin 3t(-9B + 15A - 14B) = 0$

$\Rightarrow$ $-23A - 15B = -9$

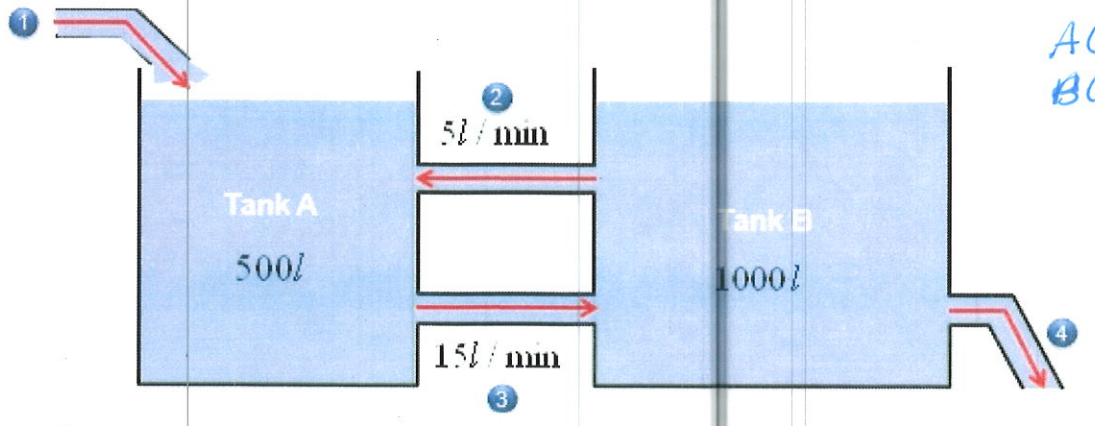
$15A - 23B = 0$

$A = \frac{207}{754}$ $B = \frac{135}{754}$

$Y_{p_2} = \frac{207}{754}\cos 3t + \frac{135}{754}\sin 3t$

$y(t) = c_1 e^{7t} + c_2 e^{-2t} - \frac{1}{5}e^{2t} + \frac{207}{754}\cos 3t + \frac{135}{754}\sin 3t$

2. A two-tank system initially starts out with 10 kg of salt in tank A and none in tank B. A 1 kg/L solution of brine is added to Tank A at a rate of 20 L/min. Water is cycled between the tanks as shown in the diagram, and then water flows out of Tank B at a rate of 20 L/min. Set up a system of differential equations that models the amount of salt in each tank. (You do not need to solve it.)



$A(0) = 10$

$B(0) = 0$

$\frac{dA}{dt} = \frac{1\text{kg}}{\text{L}} \cdot \frac{20\text{L}}{\text{min}} - \frac{A}{500-10t} \cdot \frac{15\text{L}}{\text{min}} + \frac{B}{1000+10t} \cdot \frac{5\text{L}}{\text{min}}$

$\frac{dB}{dt} = \frac{A}{500-10t} \cdot \frac{15\text{L}}{\text{min}} - \frac{B}{1000+10t} \cdot \frac{5\text{L}}{\text{min}} - \frac{B}{1000+10t} \cdot \frac{20\text{L}}{\text{min}}$

$\Rightarrow$ $\frac{dA}{dt} = 20 - \frac{15A}{500-10t} + \frac{5B}{1000+10t}$

$\frac{dB}{dt} = \frac{15A}{500-10t} - \frac{25B}{1000+10t}$