MTH 267, Quiz #11, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Solve the following systems of differential equations. Write the general solution for each in real-valued terms, and then plot the eigenvectors (when real) and give several sample trajectories.

   a. $\vec{x}'(t) = \begin{pmatrix} -7 & 6 \\ -3 & 2 \end{pmatrix}\vec{x}$

$(-7-\lambda)(2-\lambda)+18=0$

$\lambda^2+5\lambda-14+18=0$

$\lambda^2+5\lambda+4=0$

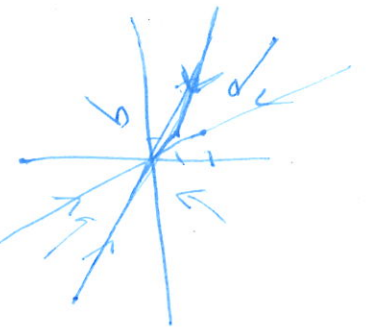
$(\lambda+4)(\lambda+1)=0$

$\lambda=-4, \lambda=-1$

$\lambda=-4$ $\begin{pmatrix} -3 & 6 \\ -3 & 6 \end{pmatrix}$ $3x_1=6x_2$ $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda=-1$ $\begin{pmatrix} -6 & 6 \\ -3 & 3 \end{pmatrix}$ $x_1=x_2$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\vec{x} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{-4t} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-t}$



   b. $\vec{x}'(t) = \begin{pmatrix} -4 & 6 \\ -3 & 2 \end{pmatrix}\vec{x}$

$(-4-\lambda)(2-\lambda)+18=0$

$\lambda^2+2\lambda-8+18=0$

$\lambda^2+2\lambda+10=0$

$\lambda = \frac{-2\pm\sqrt{4-40}}{2} = -1\pm 3i$

$\begin{pmatrix} -3-3i & 6 \\ -3 & 3-3i \end{pmatrix}$ $\frac{-3}{-3}x_1 = \frac{-(3-3i)}{+3}x_2$

$x_1 = (1-i)x_2$

$\begin{pmatrix} 1-i \\ 1 \end{pmatrix}$

$e^{-t}\begin{pmatrix} 1-i \\ 1 \end{pmatrix}(\cos 3t + i\sin 3t)$

$= e^{-t}\begin{pmatrix} \cos 3t + i\sin 3t - i\cos 3t + \sin 3t \\ \cos 3t + i\sin 3t \end{pmatrix}$

$\vec{x} = c_1 e^{-t}\begin{pmatrix} \cos 3t + \sin 3t \\ \cos 3t \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} \sin 3t - \cos 3t \\ \sin 3t \end{pmatrix}$

2. Solve the second order problem $y'' - 7y' - 18y = 0$ for the general solution using the characteristic equation.

$r^2 - 7r - 18 = 0$

$(r-9)(r+2)=0$

$r=9, r=-2$

$y = c_1 e^{9t} + c_2 e^{-2t}$