MTH 267, Quiz #10, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Solve the system $\vec{x}' = \begin{pmatrix} -7 & 6 \\ -3 & -1 \end{pmatrix}\vec{x}$ for the general solution.

$(-7-\lambda)(-1-\lambda)+18=0$

$\lambda^2+8\lambda+7+18=0$

$\lambda^2+8\lambda+25=0$

$\lambda = \frac{-8 \pm \sqrt{64-100}}{2}$

$= -4 \pm 3i$

$\begin{pmatrix} -7+4-3i & 6 \\ -3 & -1+4-3i \end{pmatrix} = \begin{pmatrix} -3-3i & 6 \\ -3 & 3-3i \end{pmatrix}$

$-3x_1 = -(3-3i)x_2$

$x_1 = \frac{(3-3i)}{+3}x_2$

$x_1 = (1-i)x_2$

$\begin{pmatrix} 1-i \\ 1 \end{pmatrix} e^{-4t}(\cos 3t + i\sin 3t) =$

$e^{-4t}\begin{pmatrix} \cos 3t + i\sin 3t - i\cos 3t + \sin 3t \\ \cos 3t + i\sin 3t \end{pmatrix}$

$\vec{x} = c_1 e^{-4t}\begin{pmatrix} \cos 3t + \sin 3t \\ \cos 3t \end{pmatrix} + c_2 e^{-4t}\begin{pmatrix} \sin 3t - \cos 3t \\ \sin 3t \end{pmatrix}$

2. A force of 50 N stretches a spring 0.8 meters. A dashpot device is attached that applies a force of 3 N for each unit of velocity. A mass of 10 kg is attached to the end of the spring and is initially released from equilibrium position with a downward velocity of 0.1 m/s. Write the second-order equation that models the system. Then convert it to a system of first-order equations. Solve the system. You may use decimals in place of square roots, but carry at least four decimal places.

$\vec{x}' = \begin{pmatrix} 0 & 1 \\ -6.25 & 0.3 \end{pmatrix}\vec{x}$ $\quad \vec{x}(0) = \begin{pmatrix} 0 \\ -0.1 \end{pmatrix}$

$\begin{pmatrix} -\lambda & 1 \\ -6.25 & .3-\lambda \end{pmatrix}$

$(-\lambda)(.3-\lambda)+6.25=0$

$\lambda^2+0.3\lambda+6.25=0$

$\lambda = \frac{-0.3 \pm \sqrt{.09-4(6.25)}}{2}$

$\lambda = -\frac{3}{20} \pm \frac{\sqrt{2491}}{20}i$

$-\frac{25}{4}x_1 = \left(-\frac{3}{20} + \frac{\sqrt{2491}}{20}i\right)x_2$

$x_1 = \frac{-4}{25}\left(-\frac{3}{20} + \frac{\sqrt{2491}}{20}i\right)x_2$

$x_1 = \left(\frac{3}{125} - \frac{\sqrt{2491}}{125}i\right)x_2$

$\begin{pmatrix} \frac{3}{20} - \frac{\sqrt{2491}}{20}i & 1 \\ -\frac{25}{4} & \frac{3}{20} - \frac{\sqrt{2491}}{20}i \end{pmatrix}$

$\begin{pmatrix} 3-\sqrt{2491}i \\ 125 \end{pmatrix}\left(e^{-\frac{3}{20}t}\right)\left(\cos\left(\frac{\sqrt{2491}}{20}t\right) + i\sin\left(\frac{\sqrt{2491}}{20}t\right)\right)$

$= e^{-\frac{3}{20}t}\begin{pmatrix} 3\cos\frac{\sqrt{2491}}{20}t + 3i\sin\frac{\sqrt{2491}}{20}t - i\sqrt{2491}\cos\frac{\sqrt{2491}}{20}t + \sqrt{2491}\sin\frac{\sqrt{2491}}{20}t \\ 125\cos\frac{\sqrt{2491}}{20}t + 125\sin\frac{\sqrt{2491}}{20}t \end{pmatrix}$

$\vec{x} = c_1 e^{-\frac{3}{20}t}\begin{pmatrix} 3\cos\frac{\sqrt{2491}}{20}t + \sqrt{2491}\sin\frac{\sqrt{2491}}{20}t \\ 125\cos\frac{\sqrt{2491}}{20}t \end{pmatrix} + c_2 e^{-\frac{3}{20}t}\begin{pmatrix} 3\sin\frac{\sqrt{2491}}{20}t - \sqrt{2491}\cos\frac{\sqrt{2491}}{20}t \\ 125\sin\frac{\sqrt{2491}}{20}t \end{pmatrix}$