MTH 266, Quiz #7, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the set of polynomials $\{1, t+t^2, 3-t^2+2t^3, -4t-t^3\}$. Does this set form a basis of $\boldsymbol{P}_3$? In other words, does the set span $\boldsymbol{P}_3$ and is the set linearly independent? Does it satisfy the definition of a subspace? [Hint: treat the coefficients of each term as entries in the 4×1 vector.]

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & -1 \end{bmatrix} \Rightarrow \text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

yes, this does form a basis for P_3 (isomorphic to $\mathbb{R}^4$)

it does form a subspace since P_n is a vector space for any n

2. Find the kernel and the column space of the matrix $A = \begin{bmatrix} 4 & -5 & 2 & 3 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$.

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 2/9 & 8/9 & 0 \\ 0 & 1 & -2/9 & 1/9 & 0 \end{bmatrix}$$

column space $= R^2 = \text{span}\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \end{bmatrix}\right\}$

kernel is null space

$$\begin{aligned} x_1 + 2/9\, x_3 + 8/9\, x_4 &= 0 \\ x_2 - 2/9\, x_3 + 1/9\, x_4 &= 0 \\ x_3 &= x_3 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2/9 \\ 2/9 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -8/9 \\ -1/9 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_5$$

$$\text{Nul}\,A = \text{span}\left\{\begin{bmatrix} -2 \\ 2 \\ 9 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ -1 \\ 0 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}$$