

KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the determinant of the matrix

$$\begin{bmatrix} 6 & 3 & 2 & -1 & 0 \\ 5 & -3 & 4 & 7 & 0 \\ 0 & 1 & -1 & -2 & 4 \\ 8 & 2 & 3 & 1 & -2 \\ -3 & 0 & 9 & -1 & -6 \end{bmatrix}$$

by any method.

See attached

2. If $\det(A) = 4$ and $\det(B) = 3$, find the values of the following expressions if A and B are both $n \times n$.

a. $\det(AB)$ 12

e. $\det(3A)$ $3^n \cdot 4$

b. $\det(A^{-1})$ $\frac{1}{4}$

f. $\det(B^{-1}AB)$ 4

c. $\det(A^5)$ $4^5 = 1024$

g. $\det(A^T)$ 4

d. $\det(-A^2B^T)$

h. $\det[(AB)^{-1}]$ $\frac{1}{12}$

$$(-1)^n (4)^2 (3)$$

$$= (-1)^n 48$$

$$\begin{pmatrix} 6 & 3 & 2 & -1 & 0 \\ 5 & -3 & 4 & 7 & 0 \\ 0 & 1 & -1 & -2 & 4 \\ 8 & 2 & 3 & 1 & -2 \\ -3 & 0 & 9 & -1 & -6 \end{pmatrix} \quad \begin{array}{l} 2R_4 + R_3 \rightarrow R_3 \\ -3R_4 + R_5 \rightarrow R_5 \end{array}$$

$$\begin{pmatrix} 6 & 3 & 2 & -1 & 0 \\ 5 & -3 & 4 & 7 & 0 \\ 16 & 5 & 5 & 0 & 0 \\ 8 & 2 & 3 & 1 & -2 \\ -27 & -6 & 0 & -4 & 0 \end{pmatrix}$$

$$-(-2) \begin{pmatrix} 6 & 3 & 2 & -1 \\ 5 & -3 & 4 & 7 \\ 16 & 5 & 5 & 0 \\ -27 & -6 & 0 & -4 \end{pmatrix} \quad \begin{array}{l} 7R_1 + R_2 \rightarrow R_2 \\ -4R_1 + R_4 \rightarrow R_4 \end{array} \quad 2 \begin{pmatrix} 6 & 3 & 2 & -1 \\ 47 & 18 & 18 & 0 \\ 16 & 5 & 5 & 0 \\ -51 & -18 & -8 & 0 \end{pmatrix}$$

$$-2(-1) \begin{pmatrix} 47 & 18 & 18 \\ 16 & 5 & 5 \\ -51 & -18 & -8 \end{pmatrix} \quad R_1 + R_3 \rightarrow R_3 \quad 2 \begin{pmatrix} 47 & 18 & 18 \\ 16 & 5 & 5 \\ -4 & 0 & 10 \end{pmatrix}$$

$$2 \left[-4 \begin{vmatrix} 18 & 18 \\ 5 & 5 \end{vmatrix} + 10 \begin{vmatrix} 47 & 18 \\ 16 & 5 \end{vmatrix} \right] = 2 \left[-4(90 - 90) + 10(235 - 288) \right]$$

$$2 \left[0 + 10(-53) \right] = -1060$$