MTH 266, Proof Set #4, Fall 2018 Name ____

Instructions: For each statement listed below write up a complete proof, following the guidelines of the proof writing handout given out in class, and the examples in the appendix in our textbook. You must include **words** explaining your reasoning, citing relevant theorems or definitions, not just computation and symbols. Proofs will be graded both on accuracy and style. Proofs may be handwritten **legibly**, or else they should be typed using a word processor equipped with an equation editor (you will asked to do a second draft of these proofs, and you may find it easier to edit if you type it up the first time). Attach the proofs to this page. You may use examples in the textbook, or those you find online, for pointers and direction, but the proofs you write should be **your own words** and **notation consistent with our textbook**.

Required: Do all the proofs in this section.

1. Show that λ is an eigenvalue of A if and only if λ is an eigenvalue of A^T. [Hint: find out how $A - \lambda I$ *and* $A^T - \lambda I$ are related.]

2. Show that if λ is an eigenvalue of A, then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

3. Prove that $A = \begin{bmatrix} 1 & -6 \\ 2 & -6 \end{bmatrix}$ is diagonalizable.

4. Show that if A is both diagonalizable and invertible, so is A^{-1}.

5. Use a counterexample to show that if A is diagonalizable, it need not be invertible.

6. Prove that if λ is an eigenvalue of A, then the matrix $A - cI$ has an eigenvalue of $\lambda - c$.

7. Prove that if A is diagonalizable with n real eigenvalues $\lambda_1, \lambda_2, \dots \lambda_n$ then $\det(A) = \prod_{i=1}^{n} \lambda_i$.

8. Prove that if λ is an eigenvalue of A, then λ^n is an eigenvalue of A^n.

Options: Do at least one proof from this section.

9. Prove that the following statements are equivalent:
   a. The determinant of a matrix A is 0.
   b. At least one eigenvalue of A is zero.
   c. A is singular.

10. Prove that if the eigenvalues of a diagonalizable matrix A are all ± 1, then $A = A^{-1}$.