MTH 266, Proof Set #1, Fall 2018 Name ____

Instructions: For each statement listed below write up a complete proof, following the guidelines of the proof writing handout given out in class, and the examples in the appendix in our textbook. You must include **words** explaining your reasoning, citing relevant theorems or definitions, not just computation and symbols. Proofs will be graded both on accuracy and style. Proofs may be handwritten **legibly**, or else they should be typed using a word processor equipped with an equation editor (you will asked to do a second draft of these proofs, and you may find it easier to edit if you type it up the first time). Attach the proofs to this page. You may use examples in the textbook, or those you find online, for pointers and direction, but the proofs you write should be **your own words** and **notation consistent with our textbook**.

Required: Do all the proofs in this section.

1. Find a condition on a, b, c, d such that the augmented matrix $\left[\begin{matrix} a & b \\ c & d \end{matrix}\middle| \begin{matrix} e \\ f \end{matrix}\right]$ has a unique solution.

2. Prove that if $AB = AC$, then B need not equal C unless A is invertible.

3. If A is $n \times n$, show that $\frac{1}{2}(A + A^T)$ is a symmetric matrix.

4. Prove that if A is idempotent, i.e. that $A^2 = A$, then either $A = I$ or A is singular.

5. Show that the algebraic properties of matrices are valid for a generic 2x2 matrices A, B, C and scalars r, s in R.
   a. $A + B = B + A$
   b. $r(A + B) = rA + rB$
   c. $(r + s)A = rA + sA$
   d. $(A^T)^T = A$
   e. $(A + B)^T = A^T + B^T$

Options: Do at least two proofs from this section.

6. Establish that for $n \times n$ matrix, $(AB)^T = B^T A^T$.

7. Establish for A is $m \times n$ and B, C are $n \times p$, that $A(B + C) = AB + AC$

8. We can define the exponential of a matrix by analogy with the definition of the exponential in terms of a Taylor series: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$, and so $e^A = \sum_{k=0}^{\infty} \frac{A^k}{k!} = I + A + \frac{A^2}{2} + \frac{A^3}{6} + \cdots$ when A is a square matrix so that the powers of A are well-defined. Show that when A is a diagonal matrix that $e^A = \begin{bmatrix} e^{a_{11}} & 0 \\ 0 & e^{a_{22}} \end{bmatrix}$ (for the 2x2 case).