MTH 266, Homework #4, Fall 2018   Name ____

Instructions: Write your work up neatly and attach to this page. Use exact values unless specifically asked to round. Show all work.

1. For each statement below determine if it is true or false. If the statement is false, briefly explain why it is false and give the true statement.
   a. If **f** is a function in the vector space V of all real-valued functions on R and if $\vec{f}(t)=0$ for some t, then **f** is the zero vector in V.

   b. A vector is an arrow in three-dimensional space.

   c. A subset H of a vector space V is a subspace of V if the zero vector is in H.

   d. A subspace is also a vector space.

   e. A vector is any element of a vector space.

   f. R^2 is a subspace of R^3.

   g. A subset H of a vector space V is a subspace of V if the following conditions are satisfied: i) the zero vector of V is in H, ii) $\vec{u}$, $\vec{v}$ and $\vec{u}+\vec{v}$ are in H, and iii) c is a scalar and $c\vec{u}$ is in H.

   h. The points in the plane corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ lie on a line through the origin.

   i. An example of a linear combination of vectors $\vec{v_1}$ and $\vec{v_2}$ is the vector $\frac{1}{2}\vec{v_1}$.

   j. Any list of 5 real numbers is a vector in R^5.

   k. If $H = span\{\vec{b_1},...,\vec{b_n}\}$, then $\{\vec{b_1},...,\vec{b_n}\}$ is a basis for H.

   l. The columns of an invertible *n*x*n* matrix form a basis for R^n.

   m. The basis is a spanning set that is as large as possible.

   n. In some cases, the linear independence relations among the columns of a matrix can be affected by certain elementary row operations of the matrix.

   o. A linearly independent set in a subspace H is a basis for H.

   p. If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.

   q. The null space of A is the solution set of the equation $A\vec{x}=\vec{0}$.

r. The null space of an *mxn* matrix is in R^m.

s. The kernel of a linear transformation is a vector space.

t. A null space is a vector space.

u. Col A is the set of all solutions of $A\vec{x} = \vec{b}$.

v. The standard method for producing a spanning set for Nul A, described previously, sometimes fails to produce a basis for Nul A.

w. If B is an echelon form of a matrix A, then the pivot columns of B for a basis for Col A.

2. Write $\vec{v} = \begin{bmatrix} 5 \\ 3 \\ -11 \\ 11 \\ 9 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ -1 \\ -1 \end{bmatrix}$. Is the solution unique?

3. For each of the sets below, determine if the set is a vector space or subspace.

a. $H = \left\{ \begin{bmatrix} a \\ b^2 \end{bmatrix}, a, b \; real \right\}$

b. $V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix}, a = b + c \right\}$

c. $T = \left\{ \begin{bmatrix} a & 2 \\ 0 & b \end{bmatrix}, a, b \; real \right\}$

d. C = $\{the\ set\ of\ all\ complex\ numbers\ of\ the\ form\ a + bi, where\ a, b\ are\ real\}$

e. $J = \{the\ set\ of\ all\ polynomials\ such\ that\ p(t) divides\ evenly\ by\ (t-1)\}$ [Hint: write $p(t)$ in factored form, with the factor $(t-1)$ pulled out. What does the other factor look like?]

f. $0 = \{the\ set\ of\ all\ odd\ functions: f(-x) = -f(x)\}$

g. W is the set of all nxn matrices such that $A^2 = A$.

h. Q is the set of all exponential functions

i. S is the set of all $n \times n$ singular matrices

4. Describe the possible echelon forms of the matrices below using 0, 1 for the pivot and * for all other entries.

a. A is a 2x2 matrix with linearly independent columns.

b. A is a 4x3 matrix, $A = \begin{bmatrix} \vec{a_1} & \vec{a_2} & \vec{a_3} \end{bmatrix}$, such that $\{\vec{a_1}, \vec{a_2}\}$ is linearly independent and $\vec{a_3}$ is not in $span\{\vec{a_1}, \vec{a_2}\}$.

c. How many pivot columns must a 6x4 matrix have if its columns are independent? Why?

5. List 5 vectors in the span of $\begin{bmatrix} 1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ 0 \\ 1 \\ -1 \end{bmatrix}$.

6. Determine if the columns of each matrix spans R^4.

a. $\begin{bmatrix} 4 & -5 & -1 & 8 \\ 3 & -7 & -4 & 2 \\ 5 & -6 & -1 & 4 \\ 9 & 1 & 10 & 7 \end{bmatrix}$

b. $\begin{bmatrix} 5 & 11 & -6 & -7 & 12 \\ -7 & -3 & -4 & 6 & -9 \\ 11 & 5 & 6 & -9 & -3 \\ -3 & 4 & -7 & 2 & 7 \end{bmatrix}$

7. For each of the sets of bases for R^3, determine which ones are linearly independent and which ones span R^3.

a. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 6 \end{bmatrix}$

c. $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 4 \end{bmatrix}$

d. $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

8. Find a basis for the space spanned by the given vectors.

a. $\begin{bmatrix} 1 \\ 0 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 10 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -6 \\ 9 \end{bmatrix}$

b. $\begin{bmatrix} -3 \\ 2 \\ 6 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -9 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -4 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ -14 \\ 0 \\ 13 \end{bmatrix}, \begin{bmatrix} -6 \\ 3 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

9. Find vectors that span the null space of the following matrices.

a. $\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 3 & -4 & -3 & 1 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$