MTH 266, Homework #1, Fall 2018 Name ____

Instructions: Write your work up neatly and attach to this page. Use exact values unless specifically asked to round. Show all work.

1. Solve the system of equations using elementary row operations on the augmented matrix for the system. State your solution in vector form, or state that the solution is inconsistent (no solution). If the solution is consistent (has a solution), state whether it is dependent (infinitely many solutions) or independent (one solution). If the system is dependent, write the solution in parametric form. (For four and five variables systems, you may use technology; two and three variable systems should be solved by hand.)

a. $\begin{cases} 3x_1 + 6x_2 = -3 \\ 5x_1 + 7x_2 = 10 \end{cases}$

b. $\begin{cases} 2x_1 - 4x_4 = -10 \\ 3x_2 + 3x_3 = 0 \\ x_3 + 4x_4 = -1 \\ -3x_1 + 2x_2 + 3x_3 + x_4 = 5 \end{cases}$

c. $\begin{cases} x_1 - 2x_2 - 3x_3 = 0 \\ x_2 + 2x_3 = 0 \\ 2x_1 - 4x_2 + 9x_3 = 0 \end{cases}$

d. $\begin{cases} 2x_1 - 6x_3 = -8 \\ x_2 + 2x_3 = 3 \\ 3x_1 + 6x_2 - 2x_3 = -4 \end{cases}$

e. $\begin{cases} 2x_1 - 5x_2 + 8x_3 = 0 \\ -3x_1 - 4x_2 + 2x_3 = 0 \end{cases}$

f. $\begin{cases} 2x_1 - 4x_4 + x_5 = 0 \\ 3x_2 + 3x_3 - x_5 = 0 \\ x_3 + 4x_4 + 6x_5 = 0 \\ -3x_1 + 2x_2 + 3x_3 + x_4 - 2x_5 = 0 \end{cases}$

2. Construct a system of equations with three variables such that the vector $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} t$ is a solution of the system.

3. Determine the value of any variables (g, h, k) in the system or augmented matrix to make the system have the indicated type of solution.

a. $\begin{cases} 4x + ky = 6 \\ kx + y = -3 \end{cases}$, infinitely many solutions

b. $\begin{cases} x + 2y + kz = 6 \\ 3x + 6y + 8z = 4 \end{cases}$, no solution

c. $\begin{bmatrix} -4 & 12 & h \\ 2 & -6 & -3 \end{bmatrix}$, consistent

d. $\begin{cases} kx + 2ky + 3kz = 4k \\ x + y + z = 0 \\ 2x - y + z = 1 \end{cases}$, exactly one solution

e. $\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix}$, consistent

f. $\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$, consistent

4. For each statement below determine if it is true or false. If the statement is false, briefly explain why it is false and give the true statement.
   a. Every elementary row operation is reversible.

   b. A 5x6 matrix has six rows.

   c. The solution set of a linear system involving variables $x_1,\ldots,x_n$ is a list of numbers ($s_1,\ldots,s_n$) that makes each equation in the system a true statement when the values $s_1,\ldots,s_n$ are substituted for $x_1,\ldots,x_n$ respectively.

   d. Two fundamental questions about a linear system involve existence and uniqueness.

   e. Two equivalent linear systems can have different solution sets.

   f. The row reduction algorithm applies only to augmented matrices for a linear system.

   g. Finding a parametric description of the solution set of a linear system is the same as *solving* the system.

   h. If one row in an echelon form of an augmented matrix is $\begin{bmatrix} 0 & 0 & 0 & 5 & 0 \end{bmatrix}$, then the associated linear system is inconsistent.

   i. If every column of an augmented matrix has a pivot, then the system is consistent.

   j. The pivot positions in a matrix depend on whether row interchanges take place.

   k. Whenever a system has free variables, the solution set contains many solutions.

   l. A homogeneous equation is always consistent.

   m. Asking whether $\vec{b}$ is in the $span\left\{\vec{a_1},\vec{a_2},\vec{a_3}\right\}$ is the same thing as asking whether the linear system corresponding to an augmented matrix $\begin{bmatrix} \vec{a_1} & \vec{a_2} & \vec{a_3} & \vec{b} \end{bmatrix}$ has a solution.

   n. The equation $A\vec{x}=\vec{b}$ is referred to as a *vector equation.*

   o. The equation $A\vec{x}=\vec{b}$ is consistent if the augmented matrix $\begin{bmatrix} A & b \end{bmatrix}$ has a pivot in every row.

   p. If the columns of an *m*x*n* matrix A span R^n, then the equation $A\vec{x}=\vec{b}$ is consistent for each $\vec{b}$ in R^m.

   q. The homogeneous equation $A\vec{x}=\vec{0}$ has the trivial solution if and only if the equation has a least one free variable.

r. The equation $\vec{x} = \vec{p} + t\vec{v}$ describes a line through $\vec{v}$ parallel to $\vec{p}$.

s. If $A\vec{x} = \vec{b}$ is consistent, then the solution of $A\vec{x} = \vec{b}$ is obtained by translating the solution set of $A\vec{x} = \vec{0}$.

t. A single vector by itself is linearly independent.

5. Row reduce the following matrices to echelon and reduced echelon form (report both versions). Explain why echelon form is not unique but reduced echelon form is.

a. $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

6. Give 5 examples of a 4x5 matrix in echelon form. Use only 0, 1, * to fill your matrix, with 1 marking the pivot points. For each matrix, indicate whether the solution is consistent or inconsistent; and if consistent whether it is independent or dependent. For dependent systems, also list the number of free variables.

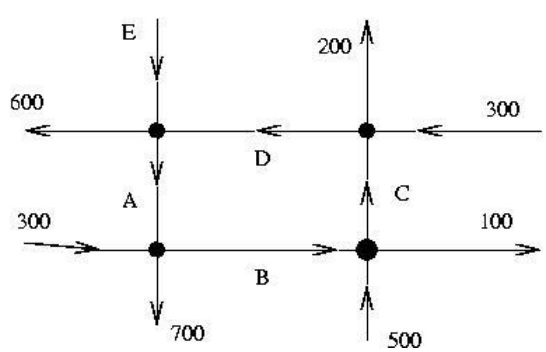
7. Find an appropriate interpolating polynomial $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots a_nx^n$ or other indicated equation for the given data. Recall that you are creating a system of equations that can be solved for the unknown coefficients. Use substitutions as appropriate.
   a. $(1,7), (2,17), (3,31), (4,65)$, cubic
   b. $(-2,28), (-1,0), (0,-6), (1,-8), (2,0)$, quartic

8. Construct a 3x3 matrix A having $a_{ij} = \frac{i}{j}$.

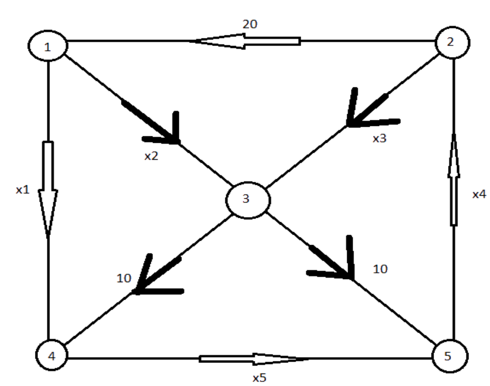
9. Construct a 3x3 matrix D having $d_{ij} = \begin{cases} i+j, & i > j \\ 0, & i = j \\ i-j, & i < j \end{cases}$.

10. For each of the traffic networks below, set up a system of equations to solve it. Solve the system with technology. Is there a single solution?

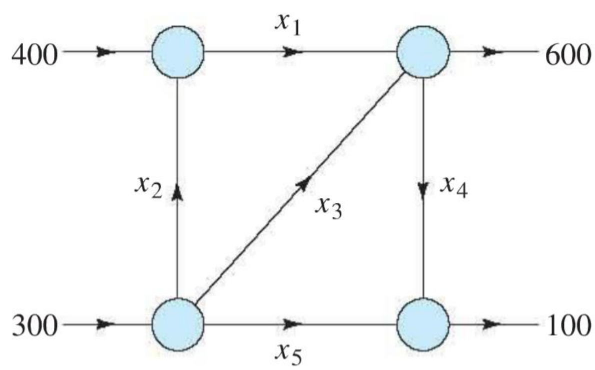
a.



c.



b.



11. Sketch the vectors $\vec{u} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \vec{v} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}, \vec{u} + \vec{v}$ and explain how this illustrates the parallelogram rule.

12. Sketch the vectors $\vec{u} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} -3 \\ 2 \\ -3 \end{bmatrix}, \vec{w} = \begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}, \vec{u} + \vec{v} + \vec{w}$.

13. For the vectors $\vec{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \vec{b} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \vec{c} = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix}, \vec{d} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \vec{e} = \begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix}$, find the following:

   a. $\vec{a} - \vec{b}$
   b. $3\vec{b} + 2\vec{a}$
   c. $\vec{c} + 2\vec{d} - 4\vec{e}$