MTH 266, Exam #1, Part I, Fall 2018 Name ____

Instructions: Show all work. Give exact answers unless specifically asked to round. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available. On this portion of the exam, you may **NOT** use a calculator.

1. Given the system of equations $\begin{cases} x_1 + 5x_2 + 5x_3 = -9 \\ -x_1 - 2x_2 - x_3 = 4 \\ \quad -3x_2 - 6x_3 = 3 \end{cases}$, write the *system* as:

   a. An augmented matrix (5 points)

   b. A vector equation (5 points)

   c. A matrix equation. (5 points)

   d. Solve the system using the augmented matrix and row operations. State whether the solution of the system is consistent or inconsistent. If the system is consistent, state whether it is independent or dependent. Write an independent solution in vector form; write a dependent solution in parametric form. (15 points)

2. Given $A = \begin{bmatrix} 2 & -2 \\ -2 & 1 \end{bmatrix}$, find A^{-1}. You can use the formula for inverses of 2x2 matrices to check your work, but use the row-reducing procedure to calculate the inverse. (8 points)

3. Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \\ 0 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -1 \\ 4 & -2 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 9 & 7 \\ 0 & 0 \end{bmatrix}$, compute the following, if possible. If the combination is not possible, briefly explain why. (6 points each)

a) AB

b) AC

c) B^T

d) 2C + A

4. Determine if $\vec{x} = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$ is a solution to the system $\begin{cases} x_1 \qquad\quad -3x_3 = 8 \\ 2x_1 + 2x_2 + 9x_3 = 7 \\ \qquad x_2 + 5x_3 = -2 \end{cases}$. (8 points)

5. Using the vectors $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$, determine the result geometrically of applying the transformation $T(\vec{x}) = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ from $R^2 \rightarrow R^2$. Sketch the before and after vectors. [Hint: Two operations are going on here. It may help to factor out an appropriate size scalar.] (15 points)

MTH 266, Exam #1, Part II, Fall 2018 Name ____

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as “all or nothing” for the answer only; partial credit will not be possible and any credit awarded for the work will not be available. On this portion of the exam, you *may* use a calculator to perform elementary matrix operations. Support your answers with work (reproduce the reduced matrices from your calculator) or other justification for full credit.

6. Determine if each statement is True or False. (2 point each)

a. T F The homogeneous system $A\mathbf{x} = \mathbf{0}$ is always consistent.

b. T F If *A* is a $m \times n$ matrix that has *m* pivot columns, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for all **b** in $\Re^m$.

c. T F Any set of vectors containing the zero vector is linearly independent.

d. T F Matrix multiplication is commutative.

e. T F A 5×7 matrix has 7 columns.

f. T F If a system of equations has a free variable then it has a unique solution.

g. T F If A is a $n \times n$ matrix, then A is invertible.

h. T F If *A* is a $m \times n$ matrix, then $AI_n = A$ and $I_m A = A$.

i. T F $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$ form a linearly dependent set.

j. T F f: $R \to R$defined by $f(x) = 2x + 3$ is a linear transformation.

7. Find the general solution to the system $\begin{cases} x_1 - 2x_2 + 4x_3 + 5x_4 = 2 \\ \quad\; x_2 - 3x_3 - x_4 = 4 \\ \quad 2x_2 - 6x_3 - 8x_4 = 6 \end{cases}$. State whether the solution of the system is consistent or inconsistent. If the system is consistent, state whether it is independent or dependent. Write an independent solution in vector form; write a dependent solution in parametric form. Circle the pivots of the reduced matrix. (15 points)

8. Determine if $\mathbf{b} = \begin{bmatrix} -7 \\ 9 \\ 14 \end{bmatrix}$ is in the span of the columns of $A = \begin{bmatrix} 1 & -4 & -14 \\ 0 & 3 & 12 \\ -2 & 8 & 28 \end{bmatrix}$.

If it is, write **b** as a linear combination of the columns of *A*; if not, explain why it is not. (12 points)

9. Let A = $\begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$

a. Determine if the columns of A form a linearly independent or dependent set and justify your answer. (6 points)

b. Determine if the columns of A span R^3. Justify your answer. (6 points)

10. Given $T: R^3 \to R^3$ such that $T(\vec{x}) = A\vec{x}$ and A is given by $A = \begin{bmatrix} 5 & 1 & -1 \\ 1 & -1 & 3 \\ 0 & 7 & 4 \end{bmatrix}$, answer the following.

a. Is T onto R^3? Justify your answer. (6 points)

b. Is T one-to-one? Justify your answer. (6 points)

11. Consider the transformation $T: P_n \to P_{n+1}$ such that $T(f) = \int_0^t f(x)dx$. If $f(x)$ is any polynomial in P_n, use the definition of a linear transformation to show that T is linear. (15 points)

12. **Use an inverse matrix** to solve $\begin{cases} x_1 \qquad\quad - 2x_3 = 1 \\ -3x_1 + x_2 + 4x_3 = -5 \\ 2x_1 - 3x_2 + 4x_3 = 8 \end{cases}$ (10 points)

13. The invertible matrix theorem states that several statements are equivalent to matrix A being invertible. Name 5 of these equivalent statements. (10 points)

14. Answer the following questions as fully as possible, and justify your answer. (6 points each)
    a. If a linear transformation $T: R^n \to R^m$ maps R^n onto R^m, can you give a relationship between m and n?

    b. If T is *one-to-one* what can you say about m and n?

    c. Explain why the columns of A^2 (defined by matrix multiplication as AA) span R^n whenever the columns of an $n \times n$ matrix A are linearly independent.

d. How many pivot columns must a 6×4 matrix have if its columns span R^4.

e. If A is a 2×5 matrix with two pivot positions, does the equation $A\vec{x} = \vec{0}$ have a solution? If so, is it trivial or non-trivial?