MTH 265, Quiz #9, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate $\int\int_R \frac{y^2}{x^2+y^2}\,dA$ where R is the region that lies between $x^2+y^2=4$ and $x^2+y^2=9$ in polar coordinates.

$$\int_0^{2\pi}\int_2^3 \frac{\cancel{r^2}\sin^2\theta}{\cancel{r^2}}\,r\,dr\,d\theta = \int_0^{2\pi}\int_2^3 r\sin^2\theta\,dr\,d\theta =$$

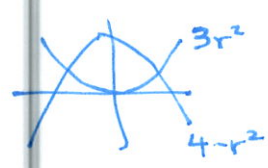
$$\frac{1}{2}r^2\Big|_2^3 \cdot \int_0^{2\pi}\sin^2\theta\,d\theta = \frac{1}{2}(9-4)\int_0^{2\pi}\frac{1}{2}(1-\cos 2\theta)\,d\theta =$$

$$\frac{5}{4}\left[\theta - \cancel{\tfrac{1}{2}\sin 2\theta}^{\,0}\right]_0^{2\pi} = \frac{5\pi}{2}$$

2. Find the volume of the solid bounded between $\overset{z}{x} = 3x^2+3y^2$ and $z = 4-x^2-y^2$. Set up a double integral in polar coordinates and evaluate it.

$3x^2+3y^2 = 4-x^2-y^2$
$4x^2+4y^2 = 4$
$x^2+y^2 = 1$

$z = 3r^2$
$z = 4-r^2$



$$\int_0^{2\pi}\int_0^1 \left[(4-r^2)-3r^2\right]r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 (4-4r^2)\,r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 4r-4r^3\,dr\,d\theta$$

$$= \int_0^{2\pi} 2r^2-r^4\Big|_0^1\,d\theta = \int_0^{2\pi} 2-1\,d\theta = \int_0^{2\pi} d\theta = 2\pi$$