

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the function $f(x, y) = \ln(x + 2y^2) - \cos xy$. Find f_x, f_y, f_{xy} .

$$f_x = \frac{1}{x+2y^2} \cdot 1 + y \sin xy = \frac{1}{x+2y^2} + y \sin xy$$

$$f_y = \frac{1}{x+2y^2} \cdot 4y + x \sin xy = \frac{4y}{x+2y^2} + x \sin xy$$

$$\begin{aligned} f_{xy} &= (-1)(x+2y^2)^{-2} \cdot 4y + \sin xy + xy \cos xy \\ &= \frac{-4y}{(x+2y^2)^2} + \sin xy + xy \cos xy \end{aligned}$$

2. Calculate the total differential of $f(x, y, z) = xe^{yz}$, and then use the values of the function at $(1, 4, 0)$ to estimate the value of the function at the point $(0.95, 4.1, 0.01)$.

$$dw = (e^{yz}) dx + (ze^{yz})(dy) + (xye^{yz})(dz)$$

$$\begin{aligned} dw &= (e^{4 \cdot 0})(0.05) + (1 \cdot 0 e^{4 \cdot 0})(0.1) + (1)(4)(e^{4 \cdot 0})(0.01) \\ &= (1)(0.05) + 0 + 4(0.01) \\ &= -0.01 \end{aligned}$$

$$f(1, 4, 0) = (1)e^{4 \cdot 0} = 1$$

$$f(0.95, 4.1, 0.01) \approx 1 - 0.01 = 0.99$$