MTH 265, Quiz #20, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the function $f(x,y) = xy - 2x - 2y - x^2 - y^2$. Find ∇f and sketch the nullclines (where f_x or f_y are equal to zero). Find the intersection(s) the nullclines and sketch one vector in each region of the plane. Use that information to determine whether the critical point(s) (intersection(s)) is (are) a maximum, minimum or saddle point. Confirm your result with technology (graph of the vector field, and a 3D graph of the original function).

$\nabla f = \langle y-2-2x,\ x-2-2y \rangle$

$y = 2x+2$

$y = -\frac{1}{2}(2-x) = \frac{1}{2}x - 1$

(x,y)	∇f
$(0,0)$	$\langle -2,-2 \rangle$
$(4,0)$	$\langle -10,2 \rangle$
$(-4,0)$	$\langle 6,-6 \rangle$
$(-3,-3)$	$\langle 1,1 \rangle$

maximum

$2x+2 = \frac{1}{2}x - 1$

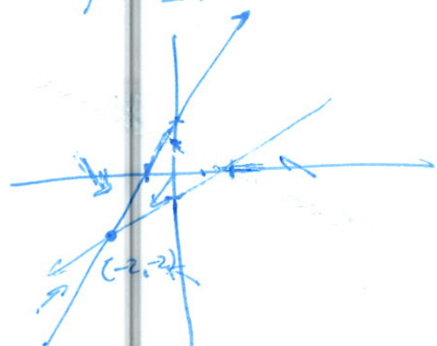
$4x+4 = x-2$

$3x = -6$

$x = -2$

$y = -2$

@ $(-2,-2)$



2. Using the same function as in #1, use the second partials test to verify the results above.

$f_x = y-2-2x$

$f_y = x-2-2y$

$f_{xx} = -2$

$f_{yy} = -2$

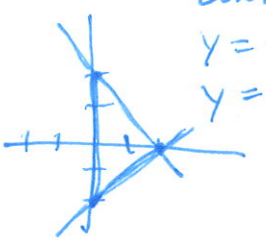
$f_{xy} = 1$

$D = (-2)(-2) - (1)^2 = 4-1 = 3$ max or min

f_{xx} ∩ ∴ maximum

@ $(-2,-2)$

3. Find the absolute extrema of the function $f(x,y) = x + y - xy$ over the region bounded by the triangle with vertices $(2,0)$, $(0,2)$, $(0,-2)$.



bounds $x=0$

$y = -x+2$

$y = x-2$

$f_x = 1-y = 0 \quad y=1$

$f_y = 1-x = 0 \quad x=1$

$f(0,y) = y \quad f'(y) = 1 \neq 0$

$f(x, x-2) = x + x - 2 - x(x-2)$

$= 2x - 2 - x^2 \cancel{-2x}$

$= -2 - x^2$

$f'(x) = -2x \rightarrow x=0$ same as corner

$f(x,-x+2) = \cancel{x} - \cancel{x} + 2 - x(-x+2)$

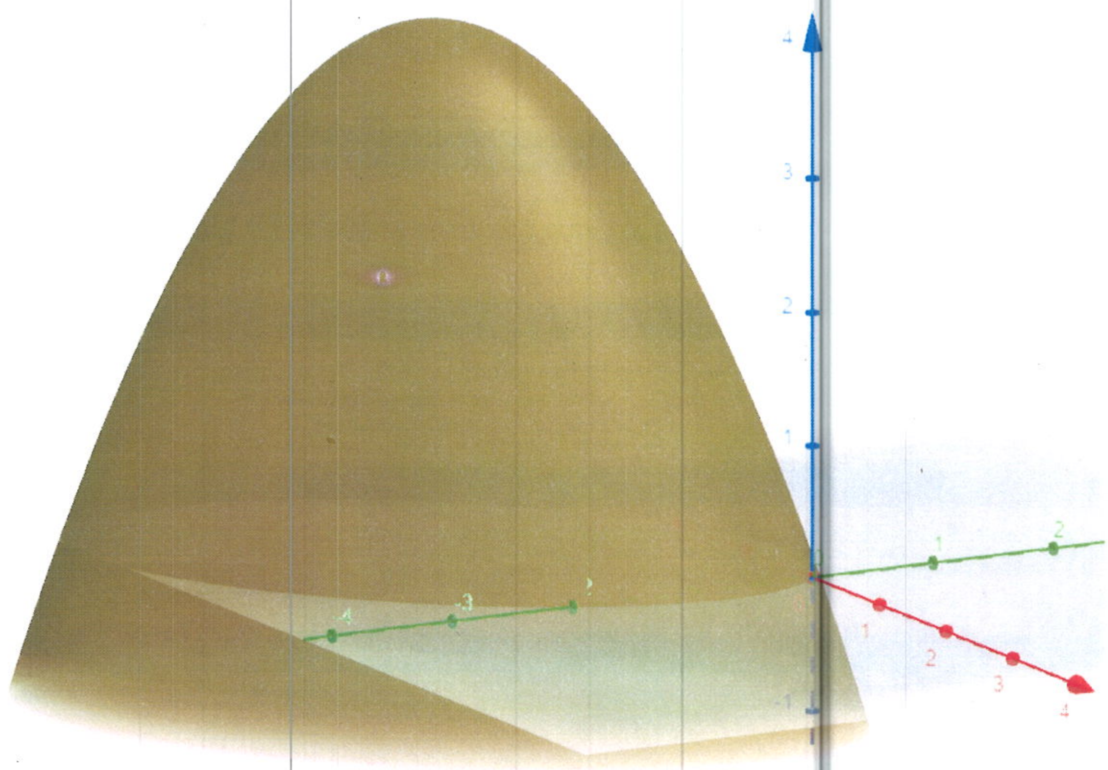
$= 2 + x^2 - 2x$

$f'(x) = 2x - 2 \rightarrow x=1 \quad y=1$ critical

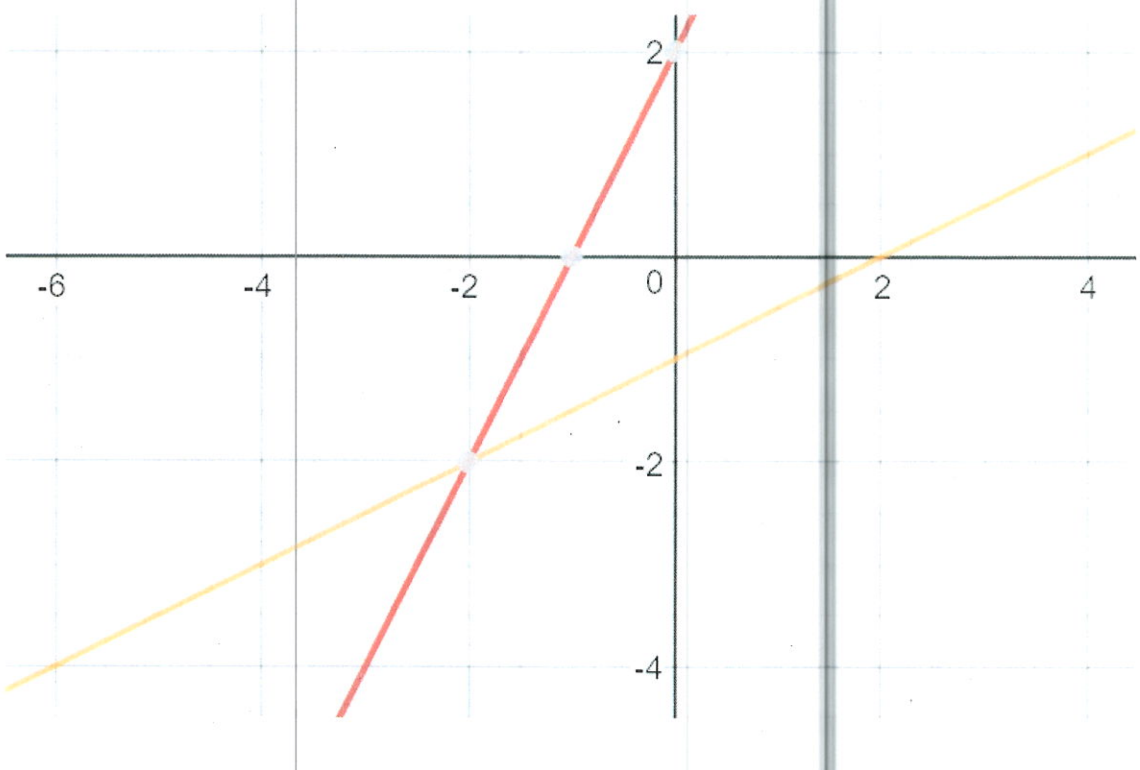
Pts	$f(x,y)$
$(1,1)$	1
$(2,0)$	2
$(0,2)$	2
$(0,-2)$	-2

Abs Max → $(2,0)$, $(0,2)$

Abs Min → $(0,-2)$



4
3
2
1
-1
1
2
-4
-3
-2
1
2
3
4



2
-6
-4
-2
0
2
4
-2
-4