MTH 265, Quiz #16, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the surface integral $\iint_S y^2\,dS$ where S is the part of the sphere $x^2+y^2+z^2=4$ that lies inside the cylinder $x^2+y^2=1$.

$G = z - \sqrt{4-x^2-y^2}$ $\qquad z = \sqrt{4-x^2-y^2}$

$\nabla G = \left\langle \frac{-x}{\sqrt{4-x^2-y^2}}, \frac{-y}{\sqrt{4-x^2-y^2}}, 1 \right\rangle \qquad \|\nabla G\| = \sqrt{\frac{x^2}{4-x^2-y^2} + \frac{y^2}{4-x^2-y^2} + \frac{1(4-x^2-y^2)}{(4-x^2-y^2)}} =$

$\int_0^{2\pi}\int_0^1 r^2\sin^2\theta \cdot \frac{2}{\sqrt{4-r^2}}\, r\,dr\,d\theta \qquad \sqrt{\frac{4}{4-x^2-y^2}} = \frac{2}{\sqrt{4-x^2-y^2}} = \frac{2}{\sqrt{4-r^2}}$

$\int_0^{2\pi}\int_0^1 \frac{2r^3\sin^2\theta}{\sqrt{4-r^2}}\,dr\,d\theta = \int_0^{2\pi} \sin^2\theta \cdot \frac{-2}{3}(x^2+8)\sqrt{4-x^2}\Big|_0^1\,d\theta =$

$\int_0^{2\pi} -\frac{2}{3}(9\sqrt{3}-16)\sin^2\theta\,d\theta = -\frac{2}{3}(9\sqrt{3}-16)\int_0^{2\pi}\frac{1}{2}(1-\cos 2\theta)\,d\theta$

$-\frac{1}{3}(9\sqrt{3}-16)\int_0^{2\pi} 1 - \cos 2\theta\,d\theta = -\frac{1}{3}(9\sqrt{3}-16)\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{2\pi} = \frac{2(9\sqrt{3}-16)\pi}{3}$

2. Find the flux $\iint_S \vec{F}\cdot d\vec{S}$ for $\vec{F}(x,y,z) = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$ where S is the surface of the paraboloid $z = 4-x^2-y^2$ that lies above the square $0 \le x \le 1, 0 \le y \le 1$ with upward orientation.

$G = 4-x^2-y^2-z$ downward

$\nabla G = \langle -2x, -2y, -1\rangle$

$F\cdot\nabla G = -2x^2y - 2y^2z - xz = -2x^2y - 2y^2(4-x^2-y^2) - x(4-x^2-y^2)$

$= -2x^2y - 8y^2 + 2x^2y^2 + 2y^4 - 4x + x^3 + xy^2$

upward $= 2x^2y + 8y^2 - 2x^2y^2 - 2y^4 + 4x - x^3 - xy^2$

$\int_0^1\int_0^1 2x^2y + 8y^2 - 2x^2y^2 - 2y^4 + 4x - x^3 - xy^2\,dy\,dx =$

$\int_0^1 -\frac{1}{4}x^4 - \frac{2}{3}x^3(y)(y-1) + 2x^2 + \frac{x^2}{2}(y^2-4) + 8xy^2\Big|_0^1\,dy$

$\int_0^1 \frac{41}{6}y^2 + \frac{2}{3}y + \frac{7}{4}\,dy = \frac{41}{18}y^3 + \frac{y^2}{2} + \frac{7}{4}y\Big|_0^1 = \frac{157}{36}$