MTH 265, Quiz #12, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the space curve $\vec{r}(t) = t\hat{\imath} + e^t\hat{\jmath} + e^{-t}\hat{k}$.
   a. Find $\vec{r}'(t)$

$r'(t) = 1\hat{\imath} + e^t\hat{\jmath} - e^{-t}\hat{k}$

   b. Find $\|\vec{r}'(t)\|$.

$\|r'(t)\| = \sqrt{1 + e^{2t} + e^{-2t}}$

   c. Are there any points at which $\|\vec{r}'(t)\|$ reaches an extremum? (minimum or maximum?)

minimum when $x=0$ $(\|r'(0)\| = \sqrt{3})$

no maximum $\to \infty$ as $x \to \infty$

   d. Find the unit tangent vector $\vec{T}(t)$.

$$\vec{T}(t) = \frac{1\hat{\imath} + e^t\hat{\jmath} - e^{-t}\hat{k}}{\sqrt{1 + e^{2t} + e^{-2t}}}$$

2. Find the unit normal vector of $\vec{r}(t) = \cos 4t\,\hat{\imath} + t\hat{\jmath} - \sin 4t\,\hat{k}$.

$r'(t) = -4\sin 4t\,\hat{\imath} + 1\hat{\jmath} - 4\cos 4t\,\hat{k}$ $\quad \|r'(t)\| = \sqrt{16\cos^2 4t + 1 + 16\sin^2 4t}$

$= \sqrt{17}$

$$T(t) = \frac{-4\sin 4t\,\hat{\imath} + 1\hat{\jmath} - 4\cos 4t\,\hat{k}}{\sqrt{17}}$$

$T'(t) = \frac{1}{\sqrt{17}}(-16\cos t\,\hat{\imath} + 0\hat{\jmath} + 16\sin 4t\,\hat{k}) = \frac{16}{\sqrt{17}}(-\cos 4t\,\hat{\imath} + \sin 4t\,\hat{k})$

$\|T'(t)\| = \frac{16}{\sqrt{17}}$

$N(t) = -\cos 4t\,\hat{\imath} + \sin 4t\,\hat{k}$

3. Find the directional derivative for the function $f(x,y) = x^2y - e^{x-y}$ at the point (1,1) in the direction of $\langle 2,-5\rangle$. $= \vec{u}$

$\nabla f = \langle 2xy - e^{x-y}, x^2 + e^{x-y}\rangle$ $\quad \nabla f(1,1) = \langle 1, 2\rangle$

$\hat{u} = \langle \frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\rangle$

$\nabla f(1,1)\cdot\hat{u} = \langle 1,2\rangle \cdot \langle \frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\rangle = \frac{2}{\sqrt{29}} - \frac{10}{\sqrt{29}} = \frac{-8}{\sqrt{29}}$