MTH 265, Exam #1, Part I, Fall 2018 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

1. Find the volume of the parallelepiped bounded by the vectors $\vec{u} = \langle 1,2,-1\rangle$, $\vec{v} = \langle 4,0,-3\rangle$, $\vec{w} = \langle -1,3,4\rangle$. (5 points)

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{vmatrix} 1 & 2 & -1 \\ 4 & 0 & -3 \\ -1 & 3 & 4 \end{vmatrix} = 1(0+9) - 2(16-3) + (-1)(12-0) =$$

$$9 - 2(13) - 12 = 9 - 26 - 12 = -29$$

$$V = \left|\vec{u}\cdot(\vec{v}\times\vec{w})\right| = |-29| = 29$$

2. Find the equation of the plane perpendicular to the line $\frac{x+2}{3} = y - 1 = \frac{z+3}{4}$, passing through the point $(2,-3,1)$. (6 points)

(x_0, y_0, z_0)

$\vec{u}_{line} = \langle 3,1,4\rangle = \vec{n}_{plane}$

plane: $3(x-2) + 1(y+3) + 4(z-1) = 0$

3. Identify the quadric surface $\frac{z^2}{4} - x^2 - \frac{y^2}{9} = 1$, and convert the equation to cylindrical and spherical coordinates. Use technology to graph the function in each coordinate system to verify your conversions. (10 points)

hyperboloid of 2 sheets

$$\frac{1}{4}\rho^2\cos^2\varphi - \rho^2\sin^2\varphi\cos^2\theta - \frac{1}{9}\rho^2\sin^2\varphi\sin^2\theta = 1$$

$$\rho^2(9\cos^2\varphi - 36\sin^2\varphi\cos^2\theta - 4\sin^2\varphi\sin^2\theta) = 36$$

$$\rho^2 = \frac{36}{9\cos^2\varphi - 36\sin^2\varphi\cos^2\theta - 4\sin^2\varphi\sin^2\theta}$$ Spherical

$$9z^2 = 36 + 36x^2 + 4y^2$$

$$9z^2 = 36 + 36r^2\cos^2\theta + 4r^2\sin^2\theta$$ Cylindrical

$$z = \pm\frac{1}{3}\sqrt{36 + 36r^2\cos^2\theta + 4r^2\sin^2\theta}$$

See attached graphs

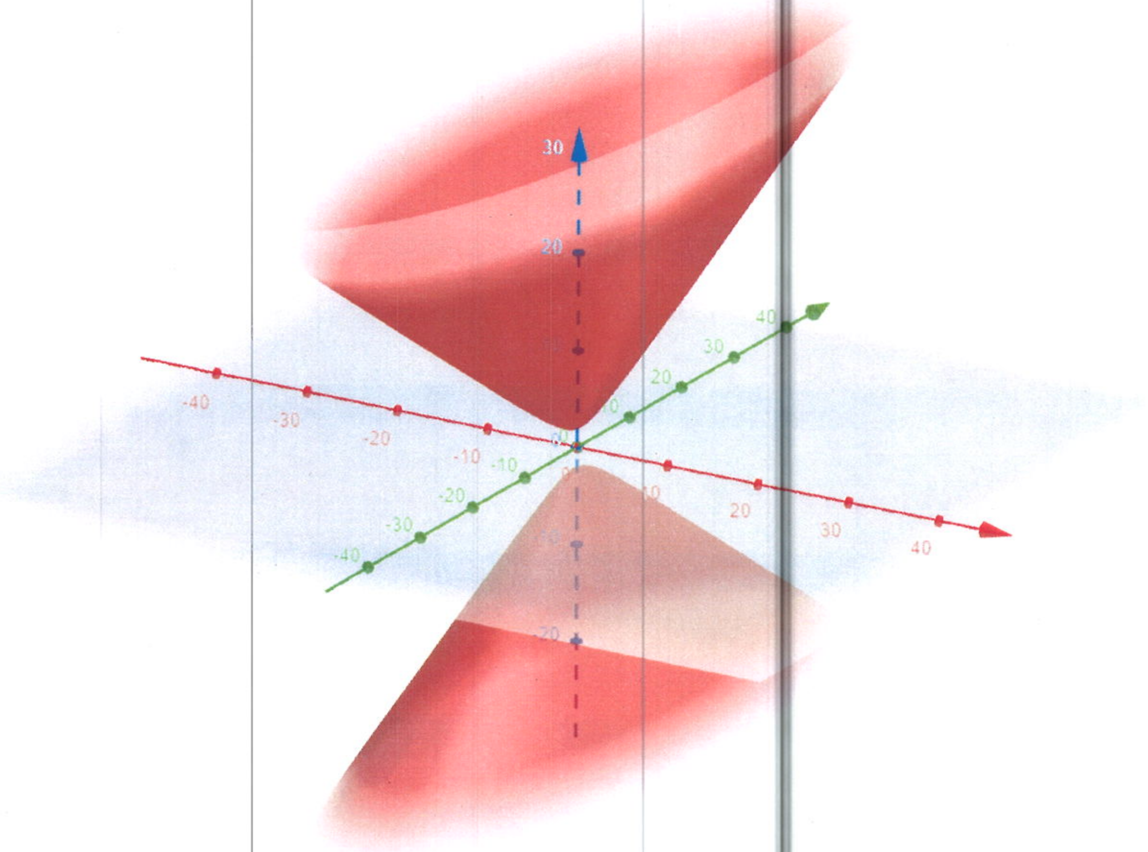
4. Write an equation of the ~~cylinder~~ cone $z = \sqrt{x^2 + y^2}$ in parametric (surface) form. Use technology to graph the original function, and the parametric form to verify your conversion. (6 points)

$$z^2 = x^2 + y^2$$

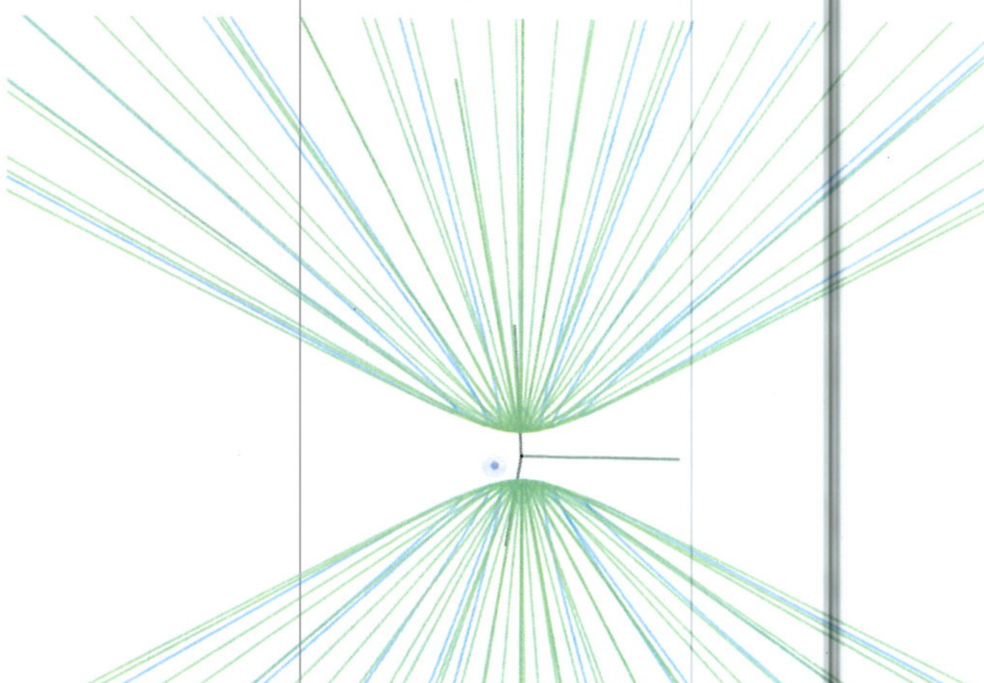
$$z = \text{radius} = v$$

$$\vec{r}(u,v) = v\cos u\,\hat{i} + v\sin u\,\hat{j} + v\,\hat{k}$$

See attached graphs

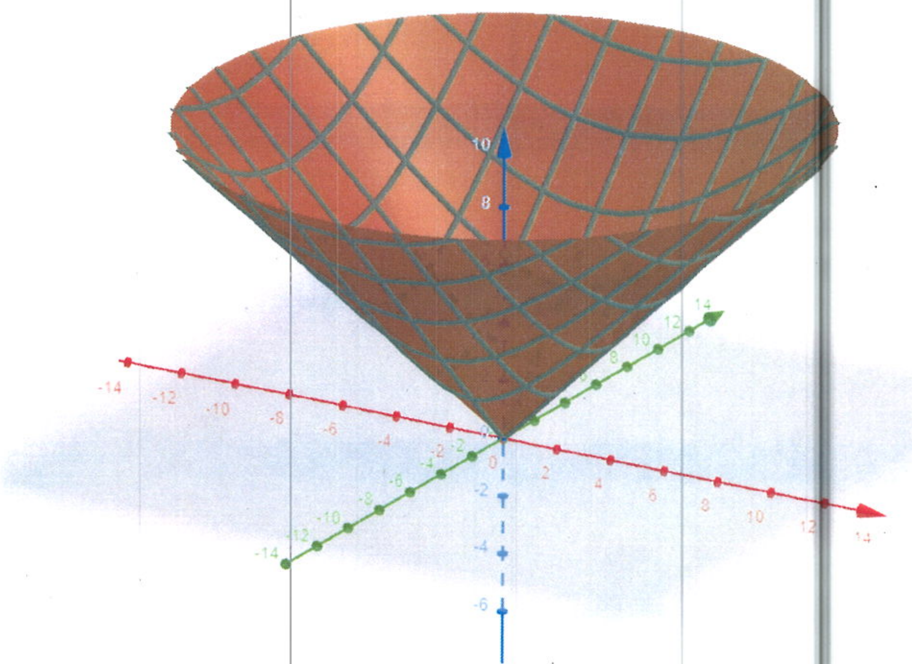


Rectangular https://www.geogebra.org/3d?lang=en



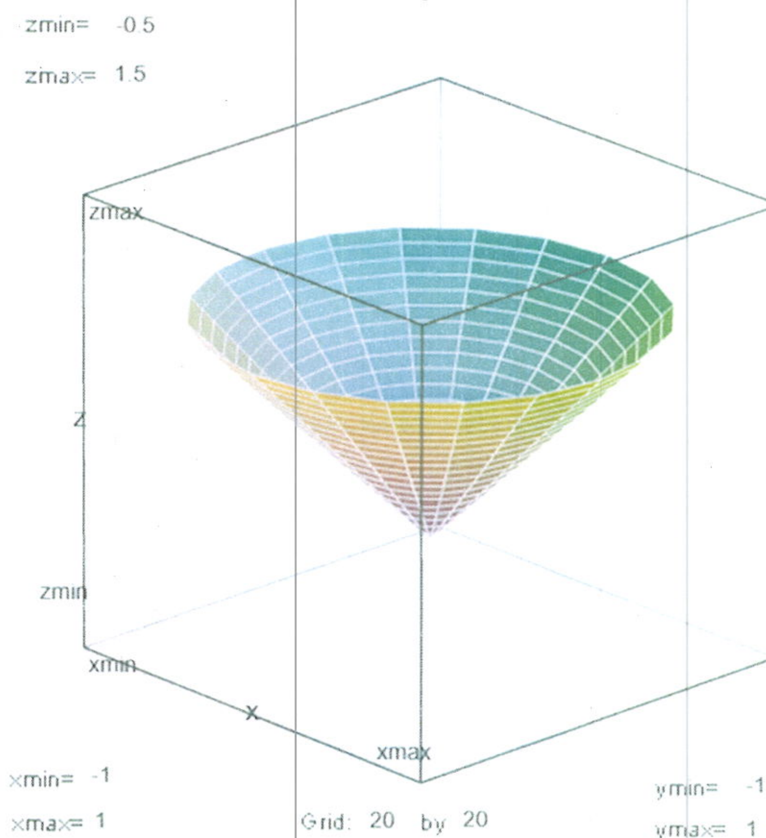
Spherical

https://www.desmos.com/calculator/fjcyqvo9zm



Rectangular

https://www.geogebra.org/3d?lang=en



Parametric Surface

http://www.math.uri.edu/~bkaskosz/flashmo/tools/parsur/

5. Use polar coordinates to find the limit if it exists or determine if it does not. (6 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy - 4y^2}{x^2 + y^2}$$

$$\lim_{r\to 0} \frac{r^2\cos\theta\sin\theta - 4r^2\sin^2\theta}{r^2} = \lim_{r\to 0} \frac{\cancel{r^2}(\cos\theta\sin\theta - 4\sin^2\theta)}{\cancel{r^2}}$$

$$= \lim_{r\to 0} \cos\theta\sin\theta - 4\sin^2\theta = \text{DNE}$$ value changes depending on θ

6. Use technology to produce a graph of the vector field $\vec{F}(x,y) = \langle y, x-y\rangle$. Include the graph with your answer, and use that graph to explain the vector displayed at the point (3,1). (8 points)

See attached

the vector at $(3,1)$ is $\langle 1, 2\rangle$ noted on graph

displayed is scaled so that it is scaled so it won't run over other vectors

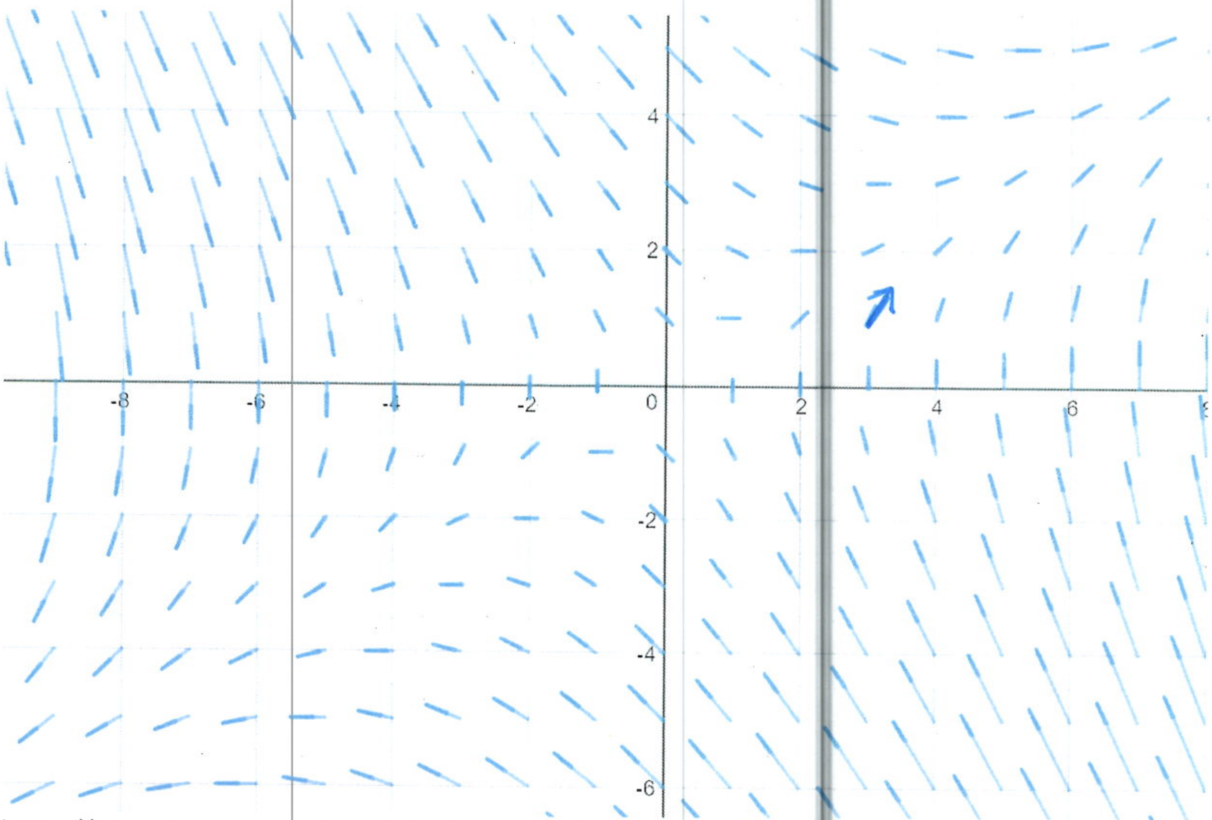
has a slope of $\frac{F_y}{F_x} = \frac{2}{1} = 2$

7. Find f_{xyz} of $f(x,y,z) = x^2y^3 + z\sin(x+y) + \sqrt{y^2 - z^2}$. (9 points)

$f_x = 2xy^3 + z\cos(x+y)$

$f_{xy} = 6xy^2 - z\sin(x+y)$

$f_{xyz} = -\sin(x+y)$



https://www.desmos.com/calculator/eijhparfmd

MTH 265, Exam #1, Part II, Fall 2018 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

8. Use $\vec{u} = \langle 2,1,-4 \rangle, \vec{v} = \langle 5,-2,3 \rangle$ to find the following. (4 points each)

   a. $\vec{u} + \vec{v}$

   $\langle 7, -1, -1 \rangle$

   b. $||\vec{v}||$

   $\sqrt{25+4+9} = \sqrt{38}$

   c. Write a unit vector in the direction of $\vec{v}$

   $\left\langle \frac{5}{\sqrt{38}}, \frac{-2}{\sqrt{38}}, \frac{3}{\sqrt{38}} \right\rangle$

   d. Find $\vec{u} \cdot \vec{v}$

   $10 - 2 - 12 = -4$

   e. Find the angle between $\vec{u}$ and $\vec{v}$ in radians, and in degrees.

   $\cos^{-1}\left(\frac{-4}{\sqrt{21}\sqrt{38}}\right) = 98.1°, \ 1.7129$ radians

   f. Find $\vec{u} \times \vec{v}$

   $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -4 \\ 5 & -2 & 3 \end{vmatrix} = (3-8)\hat{i} - (6+20)\hat{j} + (-4-5)\hat{k}$

   $\langle -5, -26, -9 \rangle$

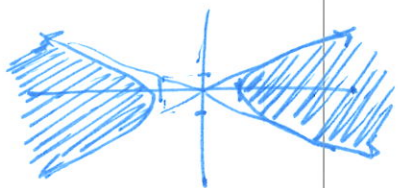
9. State the domain and range of the function $f(x,y) = \sqrt{1 - x^2 + 4y^2}$ in appropriate notation. (6 points)

$1 - x^2 + 4y^2 \geq 0$

$x^2 - 4y^2 \leq 1$

D: $\{(x,y) \mid x^2 - 4y^2 \leq 1\}$

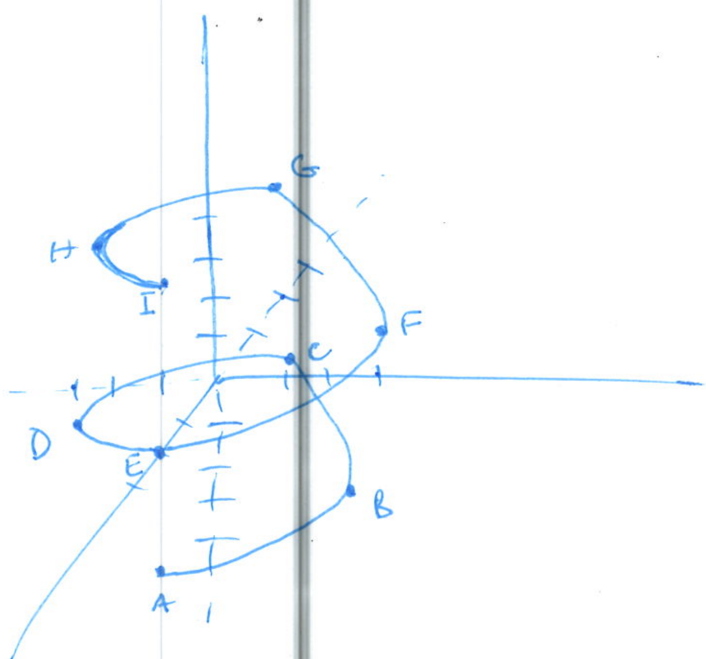
R: $0 \leq z$ or $z \geq 0$, $[0, \infty)$



10. Sketch the graph of $\vec{r}(t) = 2\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath} + \frac{t}{4}\hat{k}$ for two cycles, using a minimum of 10 points. (10 points)

≈9

t	x	y	z		
-2π	2	0	$-\pi/2$	≈ -1.6	A
$-3\pi/2$	0	3	$-3\pi/8$	≈ -1.2	B
$-\pi$	-2	0	$-\pi/4$	$\approx -.8$	C
$-\pi/2$	0	-3	$-\pi/8$	$\approx -.4$	D
0	2	0	0		E
$\pi/2$	0	3	$\pi/8$	$\approx .4$	F
π	-2	0	$\pi/4$	$\approx .8$	G
$3\pi/2$	0	-3	$3\pi/8$	≈ 1.2	H
2π	2	0	$\pi/2$	≈ 1.6	I



14. Find the value of the line integral $\int_C (x^2y^3 - \sqrt{x})ds$ on the path following the curve $y = \sqrt{x}$ between the points (1,1) and (4,2). (10 points)

$r(t) = t\hat{i} + \sqrt{t}\hat{j}$

$r'(t) = 1\hat{i} + \frac{1}{2\sqrt{t}}\hat{j}$

$ds = \sqrt{1 + \frac{1}{4t}}\, dt$

$$\int_1^4 \left(t^2 t^{3/2} - t^{1/2}\right)\sqrt{1 + \frac{1}{4t}}\, dt$$

$$\int_1^4 \left(t^{7/2} - t^{1/2}\right)\sqrt{\frac{4t+1}{4t}}\, dt =$$

$$\frac{1}{2}\int_1^4 \frac{\left(t^{7/2} - t^{1/2}\right)\sqrt{4t+1}}{t^{1/2}}\, dt = \frac{1}{2}\int_1^4 (t^3 - 1)\sqrt{4t+1}\, dt$$

$$= \frac{3015\sqrt{15}}{56} + \frac{81\sqrt{3}}{280} \approx 209.02$$

15. Find the total differential of $w = x^2yz^{1/3} + \ln yz$. Use the value of the function at the point $(1, 2, 8)$ to estimate the value of the function at $(1.05, 1.9, 9)$ (7 points)

$$dw = \left(2xyz^{1/3}\right)\Delta x + \left(x^2z^{1/3} + \frac{1}{y}\right)(\Delta y) + \left(\frac{1}{3}x^2yz^{-2/3} + \frac{1}{z}\right)\Delta z$$

$$dw = (2 \cdot 1 \cdot 2 \cdot 2)(0.05) + \left(1^2 \cdot 2 + \frac{1}{2}\right)(-0.1) + \left(\frac{1}{3} \cdot 1 \cdot 2 \cdot \frac{1}{4} + \frac{1}{8}\right)(1)$$

$$= 8(0.05) + \left(\frac{5}{2}\right)(-0.1) + \frac{7}{24}(1)$$

$$= \frac{53}{120}$$

$$w(1,2,8) = (1)^2(2)\sqrt[3]{8} + \ln(16) = 4 + \ln 16 \approx 6.77258872$$

$$w(1.05, 1.9, 9) \approx 6.77258872 + 0.441666\ldots$$

$$\approx 4 + \ln 16 + \frac{53}{120} \approx \boxed{7.214\ldots}$$