MTH 291, Skill Problems #4, Fall 2017 Name KEY

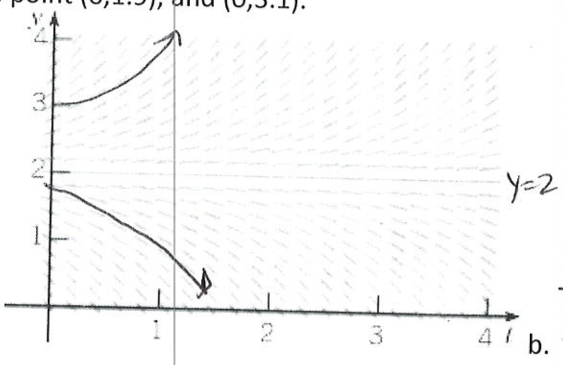
Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

1. Determine the interval on which each differential equation (together with its initial value) has a unique solution. [Hint: if $y' = f(t, y)$ check for continuity in both f *and* $\frac{\partial f}{\partial y}$.] It's possible that the solution will be a region in the ty-plane rather than an interval. In such a case, provide a graph of the region.
   a. $(t-3)y' + (\ln t)y = 2t,\ y(1) = 2$
   b. $y' = (1 - t^2 - y^2)^{1/2}$
   c. $(4 - t^2)y' + 2ty = 3t^2, y(-3) = 1$
   d. $\frac{dy}{dt} = \frac{(\cot t)y}{1+y}$

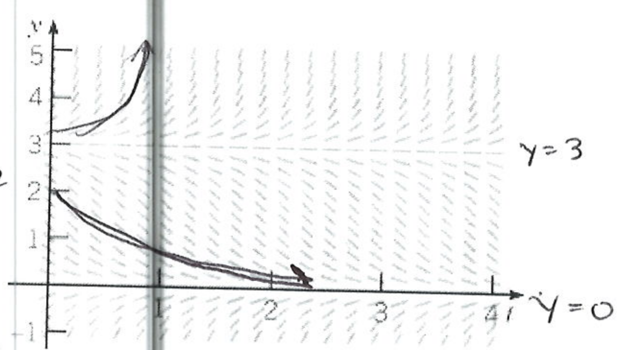
2. Determine the intervals on which the solutions are sure to exist.
   a. $y^{IV} + 4y''' + 3y = t$
   $y''' + ty'' + t^2y' + t^3y = \ln t$

3. Graph the direction field for each autonomous equation by hand and comment on the stability of each equilibrium.
   a. $y' = 1 + 2y$
   b. $y' = -y(5 - y)$
   c. $y' = y(y-2)^2$

4. For the direction fields below, state the differential equation that produces the graph (assume the equilibria are integer values). Plot the path a particle would take in the field if it started at the point (0,1.9), and (0,3.1).



a.



b.

5. A certain college graduate borrows \$8000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate k, determine the payment rate k that is required to pay off the loan in 3 years. Also determine how much interest is paid during the three-year period.

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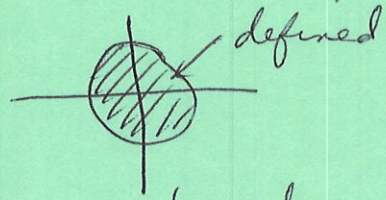
1. a. $(t-3)y' + \ln(t)y = 2t \quad y(1)=2$

defined for $t>0$, and $t \neq 3$ this solution on $(0,3)$

b. $y' = (1-t^2-y^2)^{1/2}$ $\quad 1-t^2-y^2 \geq 0$

$1 \geq t^2+y^2$

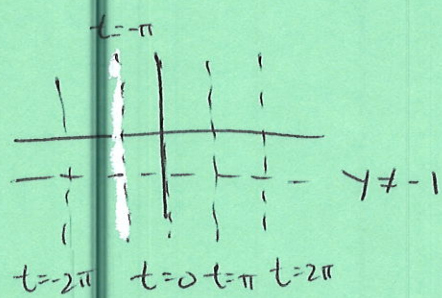


$\frac{df}{dy} = \frac{1}{2}(1-t^2-y^2)^{-1/2}(-2y) \quad \rightarrow 1 > t^2+y^2$ not on boundary

c. $(4-t^2)y' + 2ty = 3t^2 \quad y(-3)=1$

$4-t^2 \neq 0 \quad t \neq 2,-2 \quad$ on $(-\infty,-2)$

d. $\frac{dy}{dt} = \frac{(\cot t)y}{1+y} \quad y+1 \neq 0$



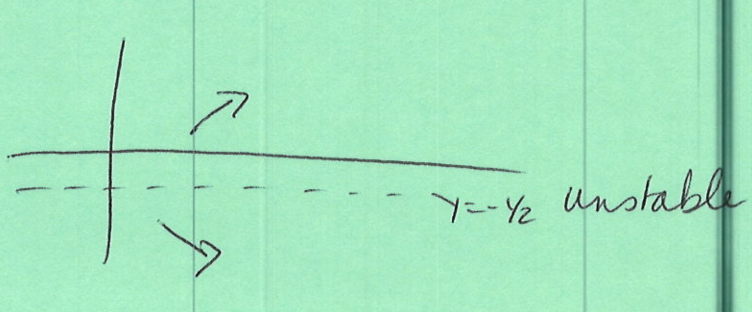
$\frac{df}{dy} = (\cot t)\left(\frac{1(1+y)-1\cdot y}{(1+y)^2}\right)$

$= (\cot t)\frac{1}{(1+y)^2}$

2. a. everywhere (all reals)

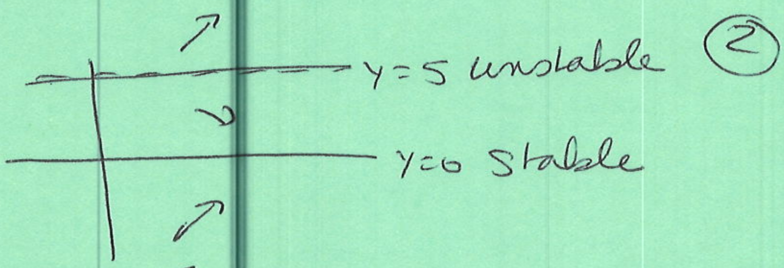
b. $t>0$

3. a. $y' = 1+2y \quad 0 = 1+2y \rightarrow -\frac{1}{2} = y$

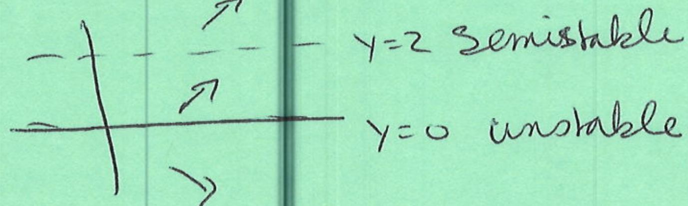


②

3b. $y' = -y(5-y)$



c. $y' = y(y-2)^2$



4a. $\frac{dy}{dt} = y-2$

b. $\frac{dy}{dt} = y(y-3)$

5. $S(0) = 8000$
$r = 10\%$
$S(3) = 0$

$\frac{dS}{dt} = rS + k$ $\quad \mu = e^{\int -.1\,dt} = e^{-.1t} = e^{-t/10}$

$\frac{dS}{dt} - .1S = k \rightarrow e^{-t/10}\frac{dS}{dt} - \frac{1}{10}e^{-t/10}S = ke^{-t/10}$

$$\int \left(e^{-t/10}S\right)' = \int ke^{-t/10}\,dt \rightarrow e^{-t/10}S = -10ke^{-t/10} + C$$

$$S = -10k + Ce^{-t/10}$$

$$8000 = -10k + C$$
$$-0 = +10k + Ce^{-3/10}$$
$$8000 = C(1+e^{-3/10})$$

$C \approx 30{,}866.37$

$$8000 = -10k + 30{,}866.37$$
$$k = -2286.64$$

payment yearly (continuous?) is 2286.64