

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the solutions to the systems.

$$a. \vec{x}' = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} \vec{x}$$

$$(2-\lambda)(3-\lambda) - 2 = 0$$

$$\lambda^2 - 5\lambda + 6 - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$(\lambda-1)(\lambda-4) = 0$$

$$\lambda = 1, 4$$

$$\lambda = 1 \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \rightarrow \begin{pmatrix} - \\ ? \end{pmatrix}$$

$$x_1 = -2x_2$$

$$\lambda = 4 \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x_1 = x_2$$

$$\vec{x} = c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$$

$$b. \vec{x}' = \begin{bmatrix} 6 & -1 \\ 5 & 2 \end{bmatrix} \vec{x}$$

$$(6-\lambda)(2-\lambda) + 5 = 0$$

$$\lambda^2 - 8\lambda + 12 + 5 = 0$$

$$\lambda^2 - 8\lambda + 17 = 0$$

$$\lambda = \frac{8 \pm \sqrt{64 - 68}}{2} = \frac{8 \pm 2i}{2} = 4 \pm i$$

$$\begin{bmatrix} 6 - (4+i) & -1 \\ 5 & 2 - (4+i) \end{bmatrix} = \begin{bmatrix} 2-i & -1 \\ 5 & -2-i \end{bmatrix}$$

$$5x_1 = (2+i)x_2 \quad \begin{pmatrix} 2+i \\ 5 \end{pmatrix}$$

$$x_1 = \frac{(2+i)}{5} x_2$$

$$e^{4t} \begin{pmatrix} 2+i \\ 5 \end{pmatrix} (\cos t + i \sin t) = e^{4t} \begin{pmatrix} 2 \cos t + 2i \sin t + i \cos t - \sin t \\ 5 \cos t + 5i \sin t \end{pmatrix}$$

$$\vec{x} = c_1 e^{4t} \begin{pmatrix} 2 \cos t - \sin t \\ 5 \cos t \end{pmatrix} + c_2 e^{4t} \begin{pmatrix} 2 \sin t + \cos t \\ 5 \sin t \end{pmatrix}$$