

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question.

1. Find the sum of $\sum_{i=0}^4 \frac{(-1)^i}{i!}$.

$$\frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!} =$$

$$\frac{1}{1} - \frac{1}{1} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} = \boxed{\frac{3}{8}}$$

2. Use mathematical induction to prove that $\sum_{i=1}^n (4i - 1) = n(2n + 1)$.

$$\text{if } n=1 \Rightarrow 4(1) - 1 = 4 - 1 = 3 \quad (1(2(1) + 1)) = 1(3) = 3 \checkmark$$

if it works for $n=k$, show for $n=k+1$

$$\sum_{i=1}^{k+1} (4i - 1) = \sum_{i=1}^k (4i - 1) + 4[(k+1)] - 1 = k(2k+1) + 4k + 3$$

$$= 2k^2 + 5k + 3 = (2k+3)(k+1) = (k+1)[2(k+1)+1]$$

which is what we expect, therefore it is true for all n .

3. Use the binomial theorem to expand $(x^2 - y)^4$.

$$\begin{aligned} & \binom{4}{0} (x^2)^4 (-y)^0 + \binom{4}{1} (x^2)^3 (-y)^1 + \binom{4}{2} (x^2)^2 (-y)^2 + \binom{4}{3} (x^2)^1 (-y)^3 + \binom{4}{4} (x^2)^0 (-y)^4 \\ & = x^8 - 4x^6y + 6x^4y^2 - 4x^2y^3 + y^4 \end{aligned}$$