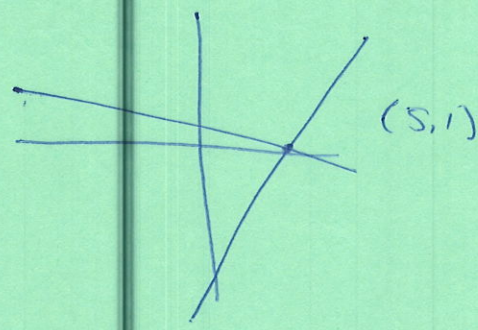


MTH 166 Homework #5 Key

1

a. $\begin{cases} x + 3y = 8 \\ y = 2x - 9 \end{cases} \rightarrow \frac{-x+8}{3} = y$



$$x + 3(2x - 9) = 8$$

$$x + 6x - 27 = 8$$

$$7x = 35$$

$$x = 5 \rightarrow y = 2(5) - 9 = 1 \quad (5, 1)$$

Consistent independent

$\begin{cases} x + 3y = 8 & \times 2 \\ -2x + y = -9 \end{cases} \rightarrow \begin{array}{r} 2x + 6y = 16 \\ -2x + y = -9 \\ \hline 7y = 7 \end{array}$

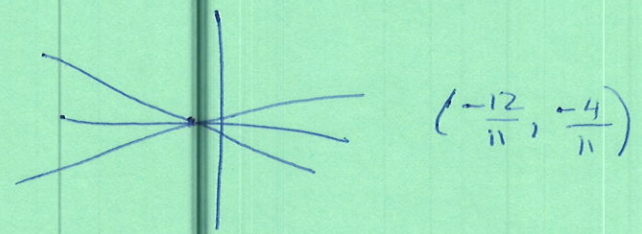
$$\frac{7y}{7} = \frac{7}{7} \rightarrow y = 1$$

$$x + 3(1) = 8$$

$$x = 5$$

$$(5, 1)$$

b. $\begin{cases} 2x + 5y = -4 \\ x - 3y = 0 \end{cases} \rightarrow \frac{-2x-4}{5} = y$
 $\frac{1}{3}x = y$



$$x = 3y$$

$$2(3y) + 5y = -4$$

$$6y + 5y = -4$$

$$11y = -4$$

$$y = -\frac{4}{11}$$

$$x = 3\left(-\frac{4}{11}\right) = -\frac{12}{11} \quad \left(-\frac{12}{11}, -\frac{4}{11}\right)$$

Consistent independent

$$\begin{array}{r} 2x + 5y = -4 \\ -2x + 6y = 0 \\ \hline 11y = -4 \end{array}$$

$$11y = -4$$

$$y = -\frac{4}{11}$$

$$x - 3\left(-\frac{4}{11}\right) = 0$$

$$x + \frac{12}{11} = 0$$

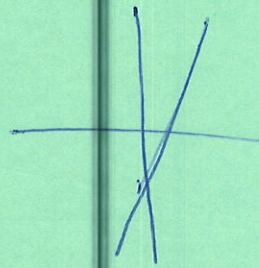
$$x = -\frac{12}{11}$$

$$\left(-\frac{12}{11}, -\frac{4}{11}\right)$$

$$1c. \begin{cases} y = 3x - 5 \\ 21x - 35 = 7y \rightarrow y = 3x - 5 \end{cases}$$

$$21x - 35 = 7(3x - 5)$$

$$21x - 35 = 21x - 35 \text{ all reals}$$



Consistent
dependent

$$y = 3x - 5 \Rightarrow -3x + y = -5 \quad \times (-7)$$

$$21x - 35 = 7y \Rightarrow 21x - 7y = 35$$

$$\underline{-21x + 7y = -35}$$

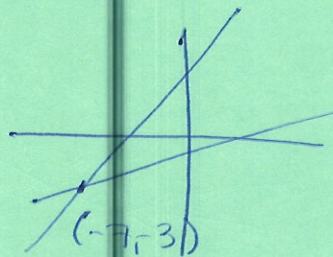
$$0 = 0$$

$$d. \begin{cases} \frac{x}{6} - \frac{y}{2} = \frac{1}{3} & *6 \\ \frac{x}{4} - \frac{y}{4} = -1 & *4 \end{cases} \rightarrow$$

$$\begin{cases} x - 3y = 2 & \rightarrow y = \frac{-x+2}{-3} \\ x - y = -4 & y = x+4 \end{cases}$$

$$\begin{array}{r} x - 3y = 2 \\ -x + y = +4 \\ \hline -2y = 6 \\ y = -3 \end{array}$$

$$\begin{array}{l} x - 3(-3) = 2 \\ x + 9 = 2 \\ x = -7 \end{array}$$



Consistent
independent

$$\begin{array}{l} x - 3y = 2 \\ x - y = -4 \rightarrow x = y - 4 \end{array}$$

$(-7, 3)$

$$\begin{array}{l} y - 4 - 3y = 2 \\ -2y - 4 = 2 \\ -2y = 6 \\ y = -3 \end{array}$$

$$x = -3 - 4 = -7$$

$(-7, 3)$

$$2. \begin{cases} \frac{x-y}{3} = \frac{x+y}{2} - \frac{1}{2} \\ \frac{x+2}{2} - 4 = \frac{x+y}{3} \end{cases} \quad (x6) \rightarrow \begin{cases} 2(x-y) = 3(x+y) - 3 \\ 3(x+2) - 24 = 2(y+4) \end{cases} \rightarrow$$

$$\begin{cases} 2x - 2y = 3x + 3y - 3 \\ 3x + 6 - 24 = 2y + 8 \end{cases} \rightarrow \begin{cases} -x - 5y = -3 \\ 3x - 2y = 26 \end{cases} \xrightarrow{\times 3} \begin{cases} -3x - 15y = -9 \\ 3x - 2y = 26 \end{cases}$$

$$\begin{aligned} -17y &= 17 \\ y &= -1 \end{aligned}$$

$$\begin{aligned} 3x - 2(-1) &= 26 \\ 3x + 2 &= 26 \\ 3x &= 24 \\ x &= 8 \end{aligned}$$

(8, -1)

$$3. \quad x + y = 200$$

$x = \# w / kitchen$
 $y = \# w / o kitchen$

$$100x + 80y = 17,000$$

$$x = 200 - y$$

$$100(200 - y) + 80y = 17,000$$

$$20,000 - 100y + 80y = 17,000$$

$$\frac{-20y}{-20} = \frac{-3000}{-20} \Rightarrow \boxed{\begin{matrix} y = 150 \\ x = 50 \end{matrix}}$$

$$4. \quad x + 5y = 6x + y$$

$$4x + 6y + y = 180$$

$$+ 5x = +4y$$

$$10x + y = 180$$

$$x = \frac{4}{5}y$$

$$10\left(\frac{4}{5}y\right) + y = 180$$

$$x = \frac{4}{5}(20) = 16$$

$$8y + y = 180$$

$$4x = 64^\circ$$

$$9y = 180$$

$$6x + y = 116^\circ$$

$$y = 20$$

$$5a. \begin{cases} 2x + y - 2z = -1 \\ 3x - 3y - z = 5 \\ x - 2y + 3z = 6 \end{cases} \xrightarrow{\begin{matrix} \times 3 \\ \times 2 \end{matrix}} \begin{cases} 6x + 3y - 6z = -3 \\ 3x - 3y - z = 5 \\ 9x - 7z = 2 \end{cases} \quad \begin{cases} 4x + 2y - 4z = -2 \\ x - 2y + 3z = 6 \\ \hline 5x - z = 4 \end{cases} \quad \textcircled{4}$$

$$\begin{cases} 9x - 7z = 2 \\ 5x - z = 4 \quad \times -7 \end{cases} \rightarrow \begin{cases} 9x - 7z = 2 \\ -35x + 7z = -28 \\ \hline -26x = -26 \\ x = 1 \end{cases} \quad \begin{matrix} 5(1) - z = 4 \\ -z = -1 \\ z = 1 \end{matrix}$$

$$2(1) + y - 2(1) = -1 \\ y = -1 \\ (1, -1, 1)$$

Consistent, independent

$$b. \begin{cases} x + z = 3 \\ x + 2y - z = 1 \\ 2x - y + z = 3 \quad \times 2 \end{cases} \rightarrow \begin{cases} x + z = 3 \\ x + 2y - z = 1 \\ 4x - 2y + 2z = 6 \end{cases} \rightarrow \begin{cases} x + z = 3 \\ -5x + z = 6 \\ \hline -4x = -3 \\ x = \frac{3}{4} \end{cases}$$

$$x + z = 3 \\ \frac{3}{4} + z = 3 \rightarrow z = \frac{9}{4}$$

$$2\left(\frac{3}{4}\right) - y + \left(-\frac{9}{4}\right) = 3 \rightarrow y = \frac{15}{4}$$

$\left(\frac{3}{4}, \frac{15}{4}, -\frac{9}{4}\right)$ consistent, independent

$$c. \begin{cases} 3(2x + y) + 5z = -1 \\ 2(x - 3y + 4z) = -9 \\ 4(1 + x) + 3 = -3(2 - 3y) \end{cases} \rightarrow \begin{cases} 6x + 3y + 5z = -1 \\ 2x - 6y + 8z = -9 \\ 4x - 9y + 3z = -7 \end{cases}$$

$$\begin{matrix} 6x + 3y + 5z = -1 \\ -6x + 18y - 24z = 27 \\ \hline 21y - 19z = 26 \end{matrix}$$

$$\begin{matrix} 4x - 9y + 3z = -7 \\ -4x + 12y - 16z = 8 \\ \hline 3y - 13z = 11 \end{matrix}$$

$$\begin{matrix} 3y - 13z = 11 \quad \times 7 \\ 21y - 19z = 26 \\ -21y + 91z = -77 \\ \hline 72z = -51 \end{matrix}$$

5

5c cont'd

$$z = -17/24$$

$$3y - 13(-17/24) = 11$$

$$3y = 11 - 221/24$$

$$3y = 43/24 \Rightarrow y = 43/72$$

$$2x - 6(43/72) + 8(-17/24) = -9$$

$$2x = 1/4$$

$$x = 1/8$$

Consistent, independent

$$(1/8, 43/72, -17/24)$$

$$d. \begin{cases} \frac{x+3}{2} - \frac{y-1}{2} + \frac{z+2}{4} = \frac{3}{2} & \times 2 \\ \frac{x-5}{2} + \frac{y+1}{3} - \frac{z}{4} = -\frac{25}{6} & \times 12 \\ \frac{x-3}{4} - \frac{y+1}{2} + \frac{z-3}{2} = -\frac{5}{2} & \times 4 \end{cases} \rightarrow \begin{cases} 2(x+3) - 2(y-1) + z+2 = 3 \\ 6(x-5) + 4(y+1) - 3z = -50 \\ x-3-2(y+1)+2(z-3) = -10 \end{cases}$$

$$\begin{cases} 2x+6-2y+2+z+2=3 \\ 6x-30+4y+4-3z=-50 \\ x-3-2y-2+2z-6=-10 \end{cases} \rightarrow \begin{cases} 2x-2y+z=-7 \\ 6x+4y-3z=-24 \\ x-2y+2z=-1 \end{cases}$$

$$\begin{array}{r} -2x+2y-2z=7 \\ x-2y+2z=1 \\ \hline -x+z=8 \end{array}$$

$$\begin{array}{r} 6x+4y-3z=-24 \\ 2x-4y+4z=2 \\ \hline 8x+z=-22 \end{array}$$

$$\begin{array}{r} x-z=-8 \\ 8x+z=-22 \\ \hline 9x=-30 \end{array}$$

$$9x = -30$$

$$x = -\frac{10}{3}$$

$$\frac{10}{3} + z = 8$$

$$z = 14/3$$

$$2(-\frac{10}{3}) - 2y + (14/3) = -7$$

$$-\frac{20}{3} + \frac{14}{3} - 2y = -7$$

$$-\frac{6}{3} - 2y = -7$$

consistent independent

$$-2y = -5$$

$$y = \frac{5}{2}$$

$$(-\frac{10}{3}, \frac{5}{2}, \frac{14}{3})$$

6

6. $ax^2 + bx + c = y$

$(-1, 6), (1, 4), (2, 9)$

$a(-1)^2 + b(-1) + c = 6$

$\rightarrow a - b + c = 6$

$\frac{-a + b - c = -6}{a + b + c = 4}$

$a(1)^2 + b(1) + c = 4$

$a + b + c = 4$

$\frac{2b = -2}{b = -1}$

$a(2)^2 + b(2) + c = 9$

$4a + 2b + c = 9$

$2b = -2$

$b = -1$

$2 + (-1) + c = 4$

$1 + c = 4$

$c = 3$

$\frac{-a + b - c = -6}{4a + 2b + c = 9}$

$\frac{3a + 3b = 3}{3a + 3(-1) = 3}$

$3a - 3 = 3$

$3a = 6$

$a = 2$

$a = 2$

$a = 2$

$2x^2 - x + 3 = y$

7. $X + Y + Z = 17000$

X = amount at 10%

$.1X + .12Y + .15Z = 2110$

Y = amount at 12%

$Y = X + Z - 1000$

Z = amount at 15%

$\begin{cases} X + Y + Z = 17,000 \\ -X + Y - Z = -1000 \\ 10X + 12Y + 15Z = 211,000 \\ -10X + 10Y + 10Z = -10,000 \end{cases} \rightarrow$

$2Y = 16000$

$Y = 8000$

$22Y + 5Z = 201,000$

$X + 8000 + 5000 = 17000$

$22(8000) + 5Z = 201,000$

$X = 4000$

$5Z = 25,000$

$(4000, 8000, 5000)$

$Z = 5,000$