MTH 166 Homework #3 Key

1a. $f(x) = -3(x+\frac{1}{2})(x-4)^3$

Zeros $-\frac{1}{2}, 4$

4 has multiplicity 3

both $-\infty, \infty$ go to $-\infty$

b. $f(x) = x^3 + 7x^2 - 4x - 28$

$x^2(x+7) - 4(x+7)$

$(x+7)(x^2-4)$

$(x+7)(x+2)(x-2)$

Zeros $-7, -2, 2$

all multiplicity 1

$-\infty \to -\infty, \infty \to \infty$

c. $f(x) = x^4 - x^2$

$x^2(x^2-1)$

$x^2(x-1)(x+1)$

Zeros $0, 1, -1$

0 multiplicity 2

both $-\infty, \infty$ go to ∞

d. $f(x) = 6x^3 - 9x - x^5$

$-x^5 + 6x^3 - 9x$

$-x(x^4 - 6x^2 + 9)$

$-x(x^2-3)^2$

Zeros $0, \pm\sqrt{3}$

$\pm\sqrt{3}$ multiplicity 2 (both)

$-\infty \to \infty, \infty \to -\infty$

e. $f(x) = x^2(x-1)^3(x+2)$

Zeros $0, 1, -2$

0 multiplicity 2, 1 multiplicity 3

2a. $a(x+2)(x-1)(x-3) = f(x)$

$f(0) = 12$

$a(2)(-1)(-3) = 12$

$6a = 12$

$a = 2$

$f(x) = 2(x+2)(x-1)(x-3)$

2b. $a(x+3)x^2(x-2)^3 = f(x)$ $f(1) = -6$

$a(1+3)(1)^2(1-2)^3 = -6$

$a(4)(1)^2(-1)^3 = -6$

$\frac{-4a}{-4} = \frac{-6}{-4} \Rightarrow a = \frac{3}{2}$

$f(x) = \frac{3}{2}(x+3)x^2(x-2)^3$

c. $f(x) = a(x+2)^3(x+1)(x-2-3i)(x-2+3i)$ $f(0) = 24$

$x^2 - 2x + 3\cancel{x}i - \cancel{2x} + 4 - \cancel{6}i - 3\cancel{x}i + \cancel{6}i + 9$

$a(x+2)^3(x+1)(x^2-4x+13)$

$a\frac{(2)^3}{8}(1)(13) = \frac{24}{8}$ $13a = 3$

$a = \frac{3}{13}$

$f(x) = \frac{3}{13}(x+2)^3(x+1)(x^2-4x+13)$

d. $f(x) = a(x+1)^2(x-1)(x-1-i)(x-1+i)(x-2-i)(x-2+i)$

$a(x+1)^2(x-1)(x^2-2x+2)(x^2-4x+5)$

$f(0) = 60$

$a(1)^2(-1)(2)(5) = 60$

$-10a = 60$

$a = -6$

$f(x) = -6(x+1)^2(x-1)(x^2-2x+2)(x^2-4x+5)$

3a.
$$\begin{array}{r} x+3 \\ x+5\overline{)x^2+8x+15} \\ -x^2 \mp 5x \\ \hline 3x+15 \\ -3x \mp 15 \\ \hline 0 \end{array}$$

long

$\underline{x+3}$

3b.

$$\begin{array}{r} 4x+3 \\ 3x-2 \overline{\big) 12x^2+x-4} \\ -12x^2 \pm 8x \\ \hline 9x-4 \\ -9x+6 \\ \hline 2 \end{array}$$

$$4x+3+\frac{2}{3x-2}$$

c.

$$\begin{array}{r} x^2+x-3 \\ x^2+x-2 \overline{\big) x^4+2x^3-4x^2-5x-6} \\ -x^4 \mp x^3 +2x^2 \\ \hline x^3-2x^2-5x-6 \\ -x^3 \mp x^2 +2x \\ \hline -3x^2-3x-6 \\ +3x^2+3x \mp 6 \\ \hline 0 \end{array}$$

$$x^2+x-3$$

d.

$$\begin{array}{r} 6x^4+12x^3+22x^2+48x+93 \\ x-2 \overline{\big) 6x^5+0x^4-2x^3+4x^2-3x+1} \\ -6x^5+12x^4 \\ \hline 12x^4-2x^3 \\ -12x^4 \mp 24x^3 \\ \hline 22x^3+4x^2 \\ -22x^3+44x^2 \\ \hline 48x^2-3x \\ -48x^2+96x \\ \hline 93x+1 \\ -93x+186 \\ \hline 187 \end{array}$$

$$6x^4+12x^3+22x^2+48x+93+\frac{187}{x-2}$$

3e.

$$\begin{array}{r}
x^6+2x^5+4x^4+8x^3+16x^2+32x+64 \\
x-2\,\big)\overline{\,x^7+0x^6+0x^5+0x^4+0x^3+0x^2+0x-128} \\
-\;x^7+2x^6 \\
\hline
2x^6 \\
-\;2x^6+4x^5 \\
\hline
4x^5 \\
-\;4x^5+8x^4 \\
\hline
8x^4 \\
-\;8x^4-116x^3 \\
\hline
16x^3 \\
-\;16x^3\mp 32x^2 \\
\hline
32x^2 \\
-\;32x^2+64x \\
\hline
64x-128 \\
-\;64x\mp 128 \\
\hline
0
\end{array}$$

$x^6+2x^5+4x^4+8x^3+16x^2+32x+64$

3a synthetic

$$\begin{array}{r|rrr}
-5 & 1 & 8 & 15 \\
 & & -5 & -15 \\
\hline
 & 1 & 3 & \lfloor 0
\end{array}$$

$x+3$ ✓

b.

$$\begin{array}{r|rrr}
\frac{2}{3} & 12 & 1 & -4 \\
 & & 8 & 6 \\
\hline
 & 12 & 9 & \lfloor +2
\end{array}$$

$3x-2=0$
$3x=2$
$x=2/3$

$12x+9+\frac{2}{x-2/3} \rightarrow 3\left(4x+3+\frac{2}{3x-2}\right)$

3d.

$$\begin{array}{r|rrrrrr} 2 & 6 & 0 & -2 & 4 & -3 & 1 \\ & & 12 & 24 & 44 & 96 & 186 \\ \hline & 6 & 12 & 22 & 48 & 93 & \boxed{187} \end{array}$$

$$6x^4+12x^3+22x^2+48x+93+\frac{187}{x-2}$$

3e.

$$\begin{array}{r|rrrrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -128 \\ & & 2 & 4 & 8 & 16 & 32 & 64 & 128 \\ \hline & 1 & 2 & 4 & 8 & 16 & 32 & 64 & \boxed{0} \end{array}$$

$$x^6+2x^4+4x^5+8x^3+16x^2+32x+64$$

4a.

$$\begin{array}{r|rrrr} 4 & 2 & -11 & 7 & -5 \\ & & 8 & -12 & -20 \\ \hline & 2 & -3 & -5 & \boxed{-25} \end{array}$$

$f(4) = -25$

b.

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 5 & 5 & -6 \\ & & 2 & -6 & -2 & 6 \\ \hline & 1 & -3 & -1 & 3 & \boxed{0} \end{array}$$

$f(2) = 0$

5a. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$

$f(x) = 3x^4 \underset{\uparrow}{-} 11x^3 - x^2 \underset{\uparrow}{+} 19x + 6$ 2 changes

$f(-x) = 3x^4 + 11x^3 - x^2 - 19x + 6$ 2 changes

> max.
2 positive zeros
2 negative zeros

56. $f(x) = x^5 \overset{1}{-} x^4 - 7x^3 \overset{2}{+} 7x^2 \overset{3}{-} 12x - 12$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

3 sign changes in $f(x) \Rightarrow$ 3 or 1 positive zero

$f(-x) = -x^5 - x^4 \underset{1}{+} 7x^3 + 7x^2 + 12x \underset{2}{-} 12$ max 2 negative zeros

c. $f(x) = x^3 \overset{1}{-} 4x^2 \overset{2}{+} 8x \overset{3}{-} 5$

$\pm 1, \pm 5$ max 3 positive zeros (or 1)

$f(-x) = -x^3 - 4x^2 - 8x - 5$ no sign changes, no negative zeros

d. $f(x) = 4x^4 \overset{1}{-} x^3 \overset{2}{+} 5x^2 \overset{3}{-} 2x - 6$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/2, \pm 3/2, \pm 1/4, \pm 3/4$

max 3 positive real zeros (or 1)

$f(-x) = 4x^4 + x^3 + 5x^2 + 2x \underset{\uparrow}{-} 6$

max one negative zero.

procedure:

if no zeros of an indicated sign can happen, Then eliminate all those rational zero.

if zeros of both signs exist, start w/ smaller (max) # of zeros and check all zeros w/ that sign until you eliminate all rational possibilities or until you find all those zeros.

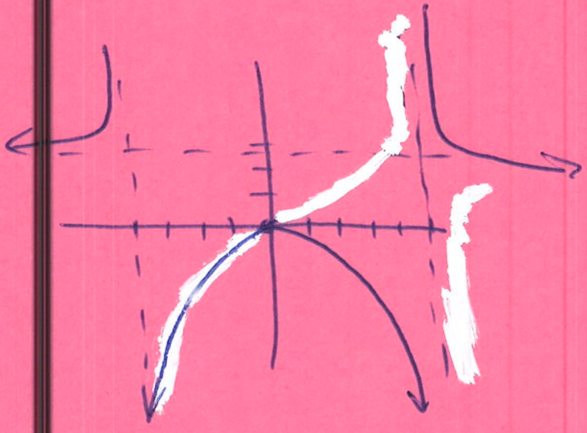
Then start on other signed zeros until the polynomial is factorable or quadratic (form) and quadratic formula can apply. (Answers will vary.)

6a. $f(x) = \frac{3x^2}{(x-5)(x+4)}$

VA: x=5, x=-4

HA: y=3

intercept x=0

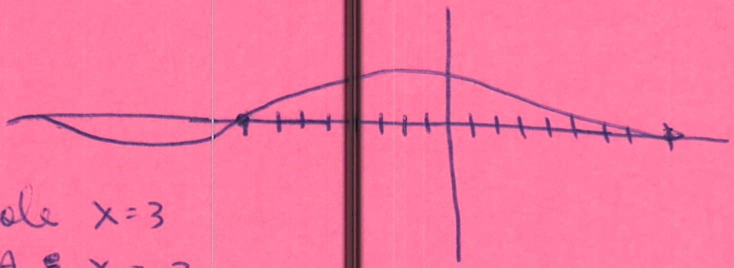


b. $g(x) = \frac{x+8}{x^2+64}$

no VA/holes

HA: 0=y

zero/intercept -8



c. $h(x) = \frac{x(x-3)}{x^2-9} = \frac{x}{x+3}$

$\sim h(3) \approx \frac{3}{6} = \frac{1}{2}$

hole x=3

VA: x=-3

HA: y=1

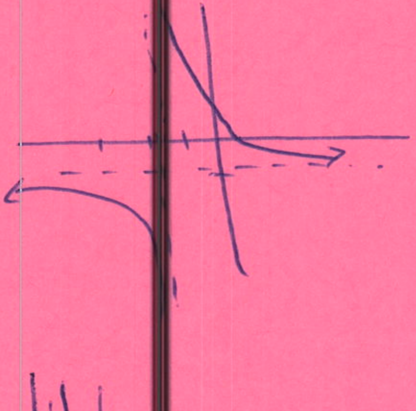
intercept x=0

d. $r(x) = \frac{-2x+1}{3x+5}$

VA: $x = -\frac{5}{3}$

HA: $y = -\frac{2}{3}$

intercept x= 1/2



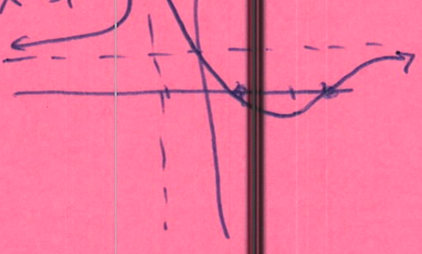
e. $q(x) = \frac{x^2-4x+3}{(x+1)^2}$

$\frac{(x-3)(x-1)}{(x+1)^2}$

VA: x=-1

HA: y=1

intercepts x=3,1



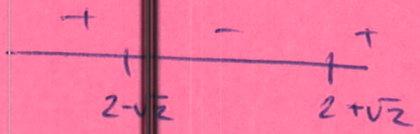
7a. $(x-7)(x+3) \geq 0$

$(-\infty,-3] \cup [7,\infty)$

b. $x^2 \leq 4x-2$

$x^2-4x+2 \leq 0$

$x = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$

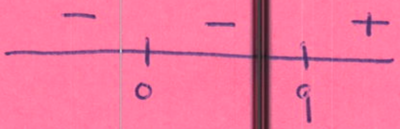


$[2-\sqrt{2}, 2+\sqrt{2}]$

c. $x^3 > 9x^2$

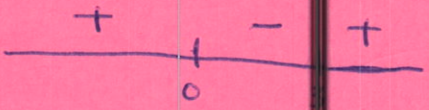
$x^3 - 9x^2 > 0$

$x^2(x-9)$



$(9,\infty)$

d. $\frac{x}{x-3} < 0$

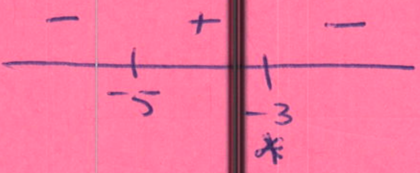


$(0,3)$

e. $\frac{x+1}{x+3} \leq 2$

$\frac{x+1}{x+3} - 2\frac{x+3}{x+3} \leq 0$

$\frac{x+1-2x-6}{x+3} = \frac{-x-5}{x+3} \leq 0$



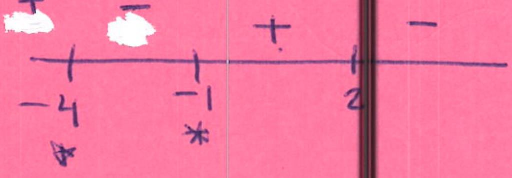
$(-\infty,-5] \cup (-3,\infty)$

f. $\frac{1}{x+1} \geq \frac{2}{x+4}$

$\frac{1}{x+1}\frac{x+4}{x+4} - \frac{2}{x+4}\frac{x+1}{x+1} \geq 0$

$\frac{x+4-2x-2}{(x+1)(x+4)} \geq 0$

$\frac{-x+2}{(x+1)(x+4)} \geq 0$



$(-\infty,-4) \cup (-1,2]$

8a. $y = kx$

$65 = k(5)$

$k = 13$

$y = 13x$

$y = 13(12) = \boxed{156}$

b. $y = \frac{k}{x}$

$12 = \frac{k}{5}$ $k = 60$

$y = \frac{60}{x}$

$y = \frac{60}{2} = \boxed{30}$

c. $y = \frac{kx}{z^2}$

$20 = \frac{k(50)}{25}$

$k = 10$

$y = \frac{10x}{\cancel{z}^2}$

$y = \frac{10(3)}{36} = \boxed{\frac{5}{6}}$

d. $y = \frac{kab}{\sqrt{c}}$

$12 = \frac{k(3)(2)}{5}$

$k = 10$

$y = \frac{10ab}{\sqrt{c}}$

$y = \frac{10(5)(\cancel{3})}{\cancel{3}} = \boxed{50}$

e. $\boxed{x = \frac{kz}{y-w}}$