MTH 166 Homework #1 Key

1a. not a function (0,-2), (0,1)

b. not a function (4,-3), (4,3) (for example)

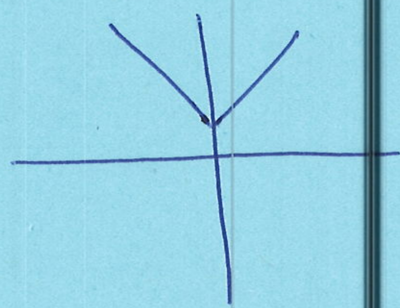
c. not a function (0,-2), (0,1)

d. function

e. function

f. function

2. $f(x) = |x| + 1$



D: $(-\infty, \infty)$

R: $[1, \infty)$

3. a. D: $(-\infty,0) \cup (0,\infty)$ i.e. $x \neq 0$

R: $(-\infty,0) \cup (0,\infty)$ i.e. $y \neq 0$

Symmetry: odd

decreasing $(-\infty,0) \cup (0,\infty)$

no extrema (relative or otherwise)

b. $f(x) = x\sqrt{1-x^2}$ $1-x^2 \geq 0 \Rightarrow 1 \geq x^2$ $x \geq -1,\ x \leq 1$

$-1 \leq x \leq 1$

D: $[-1,1]$

R: $[-\tfrac{1}{2}, \tfrac{1}{2}]$

Symmetry: odd

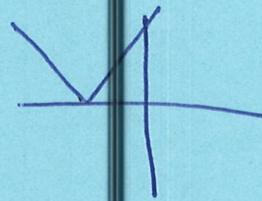
decreasing $[-1, -\tfrac{1}{\sqrt{2}}) \cup (\tfrac{1}{\sqrt{2}}, 1)$

increasing $(-\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}})$

relative min at $(-\tfrac{1}{\sqrt{2}}, -\tfrac{1}{2})$

relative max at $(\tfrac{1}{\sqrt{2}}, \tfrac{1}{2})$

3c. $f(x) = \begin{cases} x+3 & x \le -3 \\ -(x+3) & x > -3 \end{cases}$



D: $(-\infty, \infty)$

R: $[0, \infty)$

Symmetry: none

decreasing $(-\infty, -3)$

increasing $(-3, \infty)$

relative min at $(-3, 0)$

d. D: $(-\infty, \infty)$

R: $(-\infty, 3]$

Symmetry: none

increasing $(-\infty, -3) \cup (1, 4)$

decreasing $(-3, 1)$

constant $(4, \infty)$

relative max $(-3, 2)$ and $[4, \infty)$ w/ value 3

relative min $(1, -2)$

e. $f(x) = \dfrac{|x^2-4|}{x^4+1}$ D: $(-\infty, \infty)$

R: $[0, 4]$

Symmetry: even

decreasing $(-\infty, -2) \cup (0, 2)$

increasing $(-2, 0) \cup (2, \infty)$

relative min $(-2, 0)$, $(2, 0)$

relative max $(0, 4)$

f. D: $[-3, 4)$ R: $[-3, 0] \cup [1, 2]$ Symmetry: none

increasing $(-3, 0) \cup (1, 4)$ constant $(0, 1)$

relative max $(0, 1)$ w/ value 2 relative min $(-3, -3)$ and $(1, 1)$

4a. $f(x) = \begin{cases} \sqrt{x-4} & x \geq 4 \\ 4-x & x < 4 \end{cases}$ See attached.

b. $f(x) = \begin{cases} -\frac{1}{2}x^2 & x < 1 \\ 2x+1 & x \geq 1 \end{cases}$ See attached

c. $f(x) = \begin{cases} -1 & x < -2 \\ x & -2 \leq x < 1 \\ x^2 & x \geq 1 \end{cases}$ See attached

5a. $(-2,-4), (1,-1)$

$$m = \frac{-1-(-4)}{1-(-2)} = \frac{3}{3} = 1$$

$y+1 = 1(x-1) \Rightarrow y+1 = x-1 \Rightarrow \boxed{y = x-2}$
$\quad -1 \quad\quad -1$

b. $y+4 = -\frac{3}{5}(x-7)$

$y+4 = -\frac{3}{5}x + \frac{21}{5}$
$-4 \quad\quad -4 \Rightarrow \frac{-20}{5}$

$$\boxed{y = -\frac{3}{5}x + \frac{1}{5}}$$

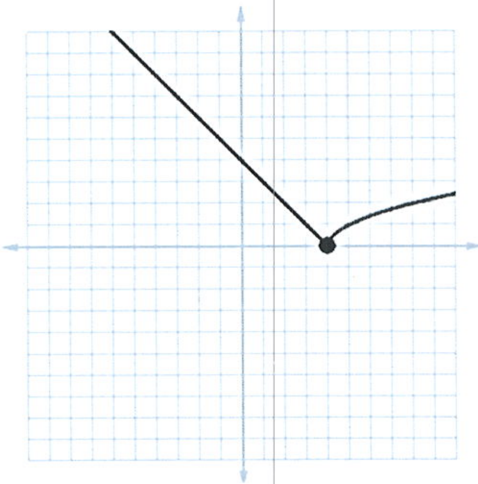
c. $y-1 = -\frac{2}{3}(x-11)$

$y-1 = -\frac{2}{3}x + \frac{22}{3}$
$+1 \quad\quad +1$

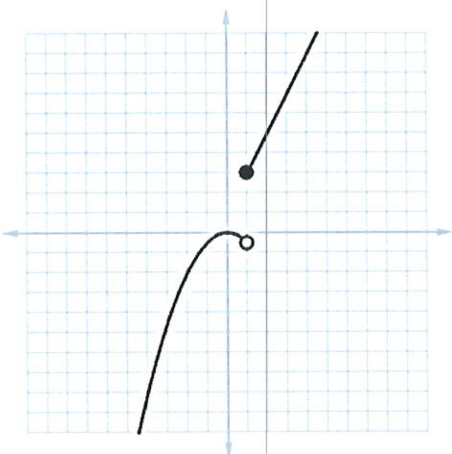
$$\boxed{y = -\frac{2}{3}x + \frac{25}{3}}$$

d. $4x-7y=14 \Rightarrow \frac{4x-14}{7} = \frac{7y}{7} \Rightarrow \frac{4}{7}x - 2 = y \quad m_\perp = \frac{-7}{4}$

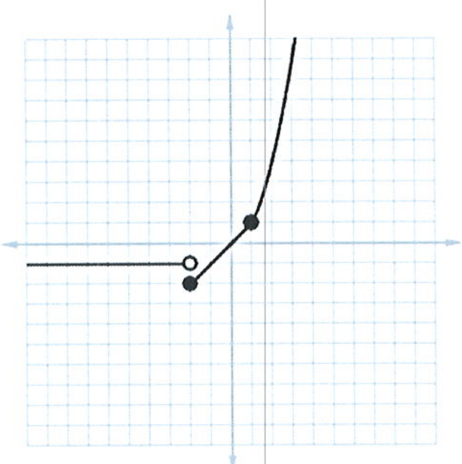
$y+4 = -\frac{7}{4}(x-8) \Rightarrow y+4 = -\frac{7}{4}x + 14 \Rightarrow \boxed{y = -\frac{7}{4}x + 10}$
$\quad -4 \quad\quad -4$



4a.



4b.



4c.

6a. $$\frac{-2(x+h)^2+(x+h)-5-(-2x^2+x-5)}{h}=$$

$$\frac{\cancel{-2x^2}-4xh-2h^2+\cancel{x}+h\cancel{-5}\cancel{+2x^2}\cancel{-x}\cancel{+5}}{h}=$$

$$\frac{-4xh-2h^2+h}{h}=\frac{\cancel{h}(-4x-2h+1)}{\cancel{h}}=\boxed{-4x-2h+1}$$

b. $$\frac{\frac{1}{x+h+2}-\frac{1}{x+2}}{h}=\frac{1}{h}\left[\frac{1}{x+h+2}-\frac{1}{x+2}\right]=\frac{1}{h}\left[\frac{x+2-(x+h+2)}{(x+h+2)(x+2)}\right]$$

$$=\frac{1}{h}\left[\frac{\cancel{x}\cancel{+2}\cancel{-x}-h\cancel{-2}}{(x+2)(x+h+2)}\right]=\frac{1}{\cancel{h}}\left[\frac{-\cancel{h}^{1}}{(x+2)(x+h+2)}\right]=\boxed{\frac{-1}{(x+h+2)(x+2)}}$$

c. $$\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}=\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}=\boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$

d. $$\frac{(x+h)^3-x^3}{h}=\frac{\cancel{x^3}+2x^2h+3xh^2+h^3\cancel{-x^3}}{h}=\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$=\boxed{3x^2+3xh+h^2}$$