MAT 166, Final Exam, Fall 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. Solve the system $\begin{cases} 5x + 12y + z = 10 \\ 2x + 5y + 2z = -1 \\ x + 2y - 3z \; = \; 5 \end{cases}$ by any method. (12 points)

Solution methods will vary

$$\left[\begin{array}{ccc|c} 5 & 12 & 1 & 10 \\ 2 & 5 & 2 & -1 \\ 1 & 2 & -3 & 5 \end{array}\right]$$

$$\Rightarrow \text{rref} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -19 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

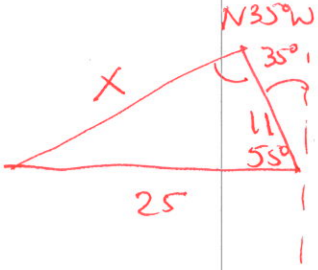
The System is inconsistent (no solution)

2. You are on a fishing boat that leaves its pier and heads east. After traveling for 25 miles, there is a report of rough seas directly north, so the captain turns the boat to a bearing of $N35°W$ for 11 miles. How far is the boat to the pier, and in which direction would the boat have to sail in order to read port from their current position? (10 points)

$X^2 = 25^2 + 11^2 - 2(25)(11)\cos 55°$

$X = 20.75$ miles

bearing $S\,44.3°\,W$



$\frac{\sin 55°}{20.75} = \frac{\sin B}{25}$

$\sin B = .98693$

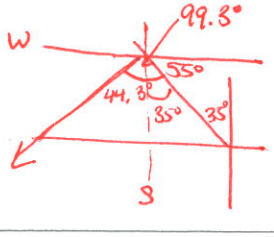
$B = 80.73°$

$\boxed{\text{or } 99.27°}$

$\cos B = \frac{25^2 - 20.75^2 - 11^2}{-2(20.75)(11)}$

$\cos B = -.16087$

$B = 99.26°$



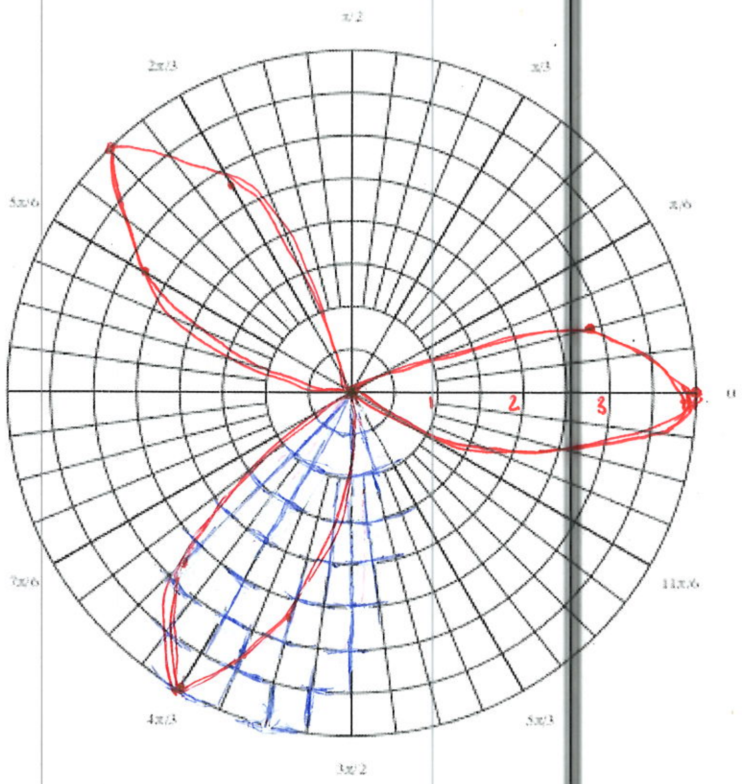
3. Convert the equation $\theta = \frac{2\pi}{3}$ into rectangular coordinates. (8 points)

$\tan^{-1}(y/x) = \theta$

$\tan\left(\frac{2\pi}{3}\right) = y/x$

$-\sqrt{3} = y/x \Rightarrow \boxed{y = -\sqrt{3}x}$

4. Graph the polar equation $r = 4\cos 3\theta$ on the polar graph below. Clearly label at least 6 points and show work. (12 points)



θ	r
0	4
$\pi/6$	0
$\pi/4$	−2.828
$\pi/3$	−4
$\pi/2$	−2.828
$2\pi/3$	0
$3\pi/4$	4
$5\pi/6$	2.828
π	0

5. Find $(-1+i)^7$ using DeMoivre's Theorem. (You will no receive credit for FOILing.) Write the result in standard form with exact values. (12 points)

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \qquad \theta = 3\pi/4$

$(\sqrt{2})^7\left(\cos 21\pi/4 + i\sin 21\pi/4\right) = (\sqrt{2})^7\left(-\frac{1}{\cancel{\sqrt{2}}} - \frac{1}{\cancel{\sqrt{2}}}i\right)$

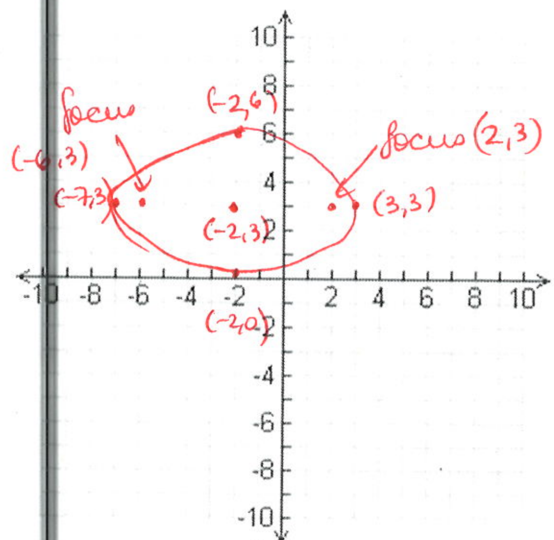
$\cancel{\sqrt{2}}\cdot(\sqrt{2})^6$

$= 8(-1-i) = \boxed{-8-8i}$

6. Graph the equation $\frac{(x+2)^2}{25}+\frac{(y-3)^2}{9}=1$ on the axes below. Clearly label the foci, vertices and minor axis endpoints. (10 points)

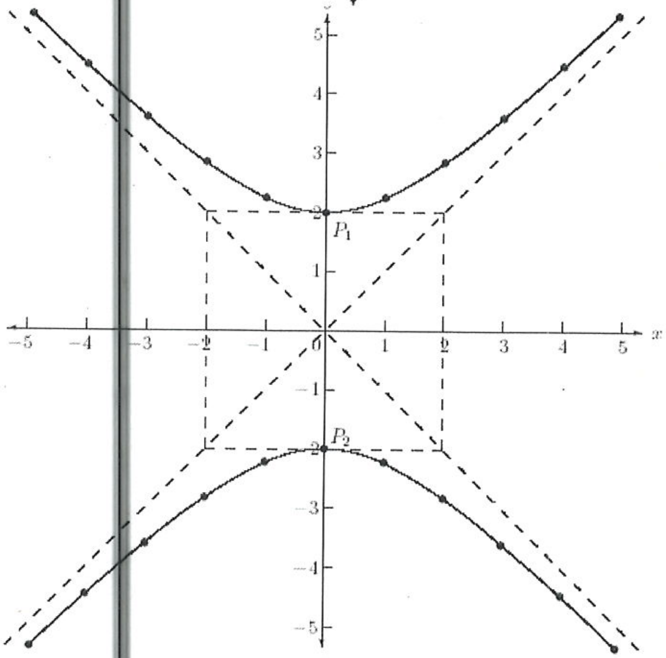
$25-9=16$

$c=4$



7. The graph of a hyperbola is shown below. Write the equation of the graph in standard form. (10 points)

$$\frac{y^2}{4}-\frac{x^2}{4}=1$$



8. Sketch the graph of the parametric equations $x=2t+3, y=-3t+1$, by plotting at least 4 points (and labeling them). Use an arrow to indicate the orientation of time. Then convert the equation back to an equation in x and y only. (10 points)

t	x	y
-3	-3	10
-2	-1	7
-1	1	4
0	3	1
1	5	-2
2	7	-5
3	9	-8

$\frac{x-3}{2}=t$

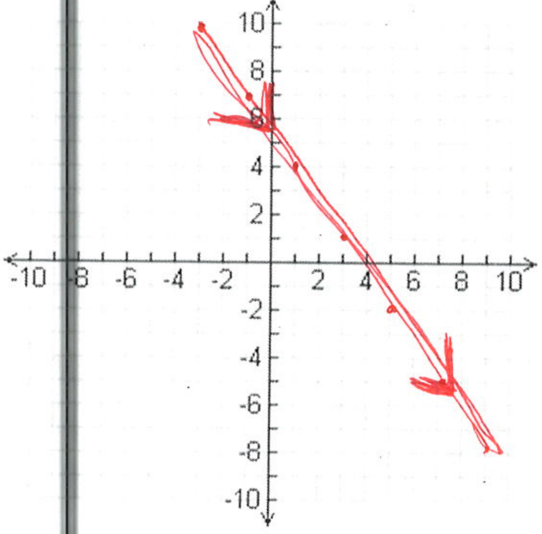
$\frac{y-1}{-3}=t$

$\frac{x-3}{2}=\frac{y-1}{-3}$

$-3(x-3)=2(y-1)$

$-3x+9=2y-2$

$-3x+11=2y$ $\Rightarrow$ $\boxed{y=-\frac{3}{2}x+\frac{11}{2}}$



9. Evaluate the following expressions. (8 points each)

a. $\frac{20!}{4!16!}$

4845

b. $\sum_{k=1}^{4}(k-3)(k+2)$

$(-2)(3) + (-1)(4) + (0)(5) + (1)(6)$

$\cancel{-6} - 4 + 0 + \cancel{6} = -4$

c. $\binom{15}{2}$

105

10. Write the sum of $1 + 8 + 27 + 64 + 125 + \cdots + 729$ in summation notation. (8 points)

$$\sum_{k=1}^{9} i^3$$

11. Use the binomial theorem to expand $(2y-3)^4$. (10 points)

$(2y)^4 + 4(2y)^3(-3) + 6(2y)^2(-3)^2 + 4(2y)(-3)^3 + (-3)^4$

$16y^4 - 96y^3 + 216y^2 - 216y + 81$

12. Find the exact value of the six trig functions if the coterminal side of the angle passes through the point $(2, -5)$. (12 points)

x y

$r = \sqrt{2^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$

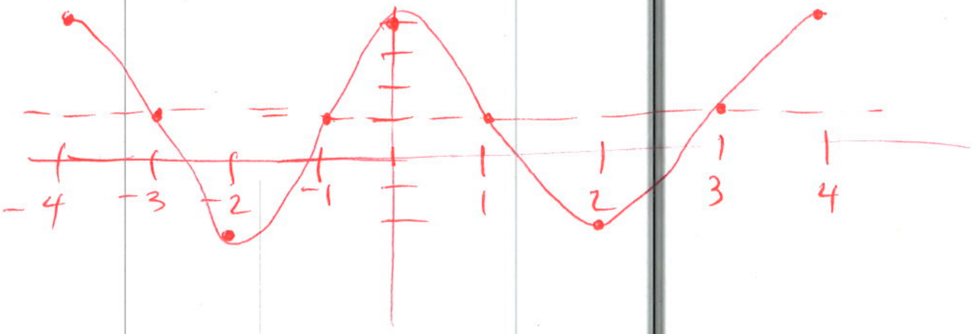
$\sin\theta = \frac{-5}{\sqrt{29}}$ $\quad\cot\theta = \frac{2}{-5}$

$\cos\theta = \frac{2}{\sqrt{29}}$ $\quad\sec\theta = \frac{\sqrt{29}}{2}$

$\tan\theta = -\frac{5}{2}$ $\quad\csc\theta = \frac{\sqrt{29}}{-5}$

13. Use key points to graph two periods of the function $y = 3\cos\frac{\pi}{2}x + 1$, by hand, using key points. (10 points)

$T = \frac{2\pi}{\pi/2} = 4$

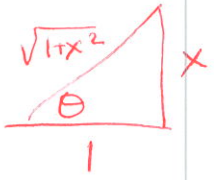


14. Find the exact value of each expression. (7 points each)

a. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$-\pi/4$

b. $\sec(tan^{-1}(x))$



$\sqrt{1+x^2}$

15. Find the domain and range of $y = -\sin^{-1}(x-1) + \frac{\pi}{4}$. (8 points)

$y = \sin^{-1} x$ $\quad D: [-\pi/2, \pi/2] \rightarrow \boxed{[-\pi/2 + 1, \pi/2 + 1]}$ D

$\quad R: [-1, 1] \rightarrow \boxed{[-1 + \pi/4, 1 + \pi/4]}$ R

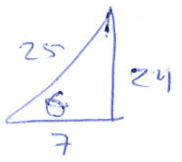
16. For $\cos\theta = \frac{7}{25}$, and θ in Q II, find each of the following. (7 points each)

a. $\cos 2\theta$

$2\cos^2\theta - 1 = 2\left(\frac{7}{25}\right)^2 - 1 = -527/625 = -.8432$

b. $\tan\frac{\theta}{2}$ $= \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{24/25}{1 + 7/25}\,\dfrac{25}{25} = \dfrac{24}{25+7} = \dfrac{24}{32}$

$= \dfrac{3}{4}$



17. Solve the equation $7\cos x - 4 = 2\sin^2 x$ for all values of x in $[0, 2\pi)$. Use exact values when possible, or round answers to 4 decimal places. (10 points)

$-2\sin^2 x + 7\cos x - 4 = 0$

$-2(1 - \cos^2 x) + 7\cos x - 4 = 0$

$2\cos^2 x - 2 + 7\cos x - 4 = 0$

$2\cos^2 x + 7\cos x - 6 = 0$

$\cos x = \dfrac{-7 \pm \sqrt{49+48}}{2}$

$\dfrac{-7 \pm \sqrt{97}}{2}$

~~$\cos x = -8.4244$~~

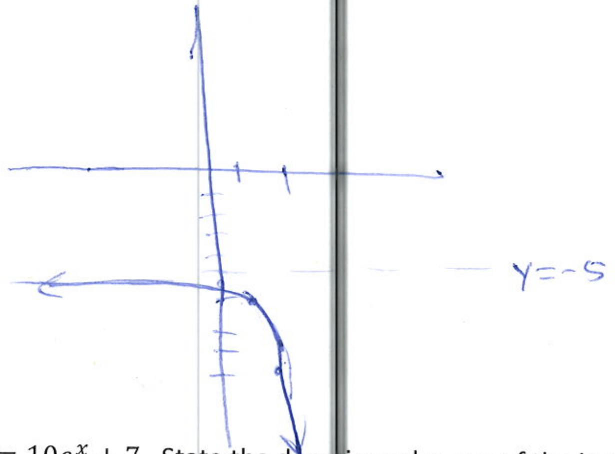
~~$\cos x = 1.4244$~~

no solutions

18. Sketch the graph of the curve $f(x) = -3^{x-1} - 5$. State the domain and range. (10 points)

D: $(-\infty, \infty)$

R: $(-\infty, -5)$



19. Find the inverse function of $f(x) = 10e^x + 7$. State the domain and range of the inverse. (8 points)

$x = 10e^y + 7$

$\frac{x-7}{10} = e^y$

$f^{-1}(x) = y = \ln\left(\frac{x-7}{10}\right)$

$\frac{x-7}{10} > 0$

$x - 7 > 0$

$x > 7$

D: $x > 7$ or $(7, \infty)$

R: $(-\infty, \infty)$

20. Solve the following equations without using a calculator. (8 points each)

a. $\log(x + 4) - \log 2 = \log(5x + 1)$

$\cancel{\log}\left(\frac{(x+4)}{2}\right) = \cancel{\log}(5x+1)$

$x + 4 = 10x + 2$

$2 = 9x$

$\boxed{x = \frac{2}{9}}$

b. $e^{2x} - 4e^x - 12 = 0$

$u = e^x$

$u^2 - 4u - 12 = 0$

$(u-6)(u+2) = 0$

$u = 6$ $u = -2$

$e^x = 6$ $\cancel{e^x = -2}$

$\boxed{x = \ln 6}$

21. Find $\frac{f(x+h)-f(x)}{h}$ for $f(x) = -2x^2 + 3x - 5$. (10 points)

$$\frac{-2(x+h)^2+3(x+h)-5-(-2x^2+3x-5)}{h} =$$

$$\frac{-2(x^2+2xh+h^2)+3x+3h-5+2x^2-3x+5}{h}$$

$$= \frac{-2\cancel{x^2}-4xh-2h^2+3\cancel{x}+3h-\cancel{5}+2\cancel{x^2}-3\cancel{x}+\cancel{5}}{h}$$

$$= \frac{-4xh-2h^2+3h}{h} = \frac{\cancel{h}(-4x-2h+3)}{\cancel{h}} = \boxed{-4x-2h+3}$$

22. Find an equation of the line with the following properties passing through the points $(-2, -5)$ and $(6, -5)$ in either rectangular coordinates or parametric form. (10 points)

$$\frac{-5-(-5)}{-2-6} = \frac{0}{-8} = 0$$

$$\boxed{y = -5}$$

parametric

$x = t$

$y = -5$

23. If $f(x) = |x|$, write the function that has all the following transformations applied: (8 points)
   a. Shift left 4 units
   b. Reflect over the x-axis
   c. Compress by a factor of 2
   d. Shift up by 3

$|x+4|$

$-|x+4|$

$-\frac{1}{2}|x+4|$

$-\frac{1}{2}|x+4|+3$

$g(x) = -\frac{1}{2}|x+4|+3$

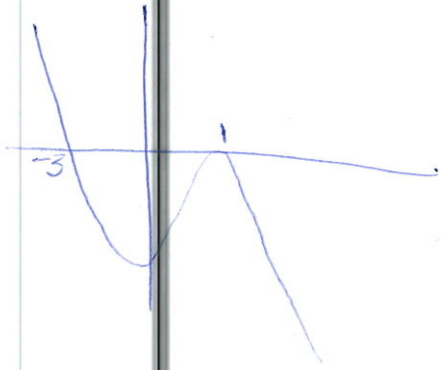
24. Given $g(x) = \sqrt{x-4}$, $h(x) = x + \frac{1}{x}$, find $(h \circ g)(x)$ and state the domain. (8 points)

$$(h \circ g) = (\sqrt{x-4}) + \frac{1}{(\sqrt{x-4})}$$

D: $x > 4$

25. Find all the possible rational zeros of the polynomial $f(x) = -x^3 - x^2 + 5x - 3$. Use them to factor the polynomial, and find all the real (and complex, if any) zeros. Write the polynomial in factored form. (10 points)

$-(x+3)(x-1)^2$

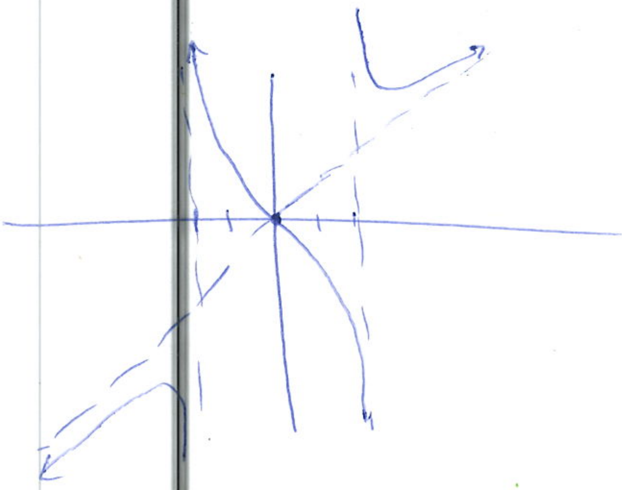


26. Sketch the graph of the function $f(x) = \frac{x^3+x}{x^2-4}$, but finding i) any intercepts, ii) any vertical asymptotes or holes, iii) any horizontal or slant asymptotes. (10 points)

$\frac{(x^2+1)x}{(x-2)(x+2)}$

$x = 0$ y · int
$x = 2, x = -2$ v. asymp
$y = x$ slant. asymp.

$$\begin{array}{r} x \\ x^2-4\,\big)\,x^3+0x^2+x+0 \\ -\;x^3+\quad 4x \\ \hline 5x \end{array}$$



Some useful formulas

$$\sin(a+b)=\sin a\cos b+\sin b\cos a$$

$$\sin(a-b)=\sin a\cos b-\sin b\cos a$$

$$\cos(a+b)=\cos a\cos b-\sin a\sin b$$

$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$

$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}$$

$$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

$$\begin{aligned}\sin 2\alpha &= 2\sin\alpha\cos\alpha\\ \cos 2\alpha &= \cos^2\alpha-\sin^2\alpha\\ &= 2\cos^2\alpha-1\\ &= 1-2\sin^2\alpha\\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha}\end{aligned}$$

$$\sin(\frac{a}{2})=\pm\sqrt{\frac{(1-\cos a)}{2}}$$

$$\cos(\frac{a}{2})=\pm\sqrt{\frac{(1+\cos a)}{2}}$$

$$\tan(\frac{a}{2})=\frac{1-\cos a}{\sin a}=\frac{\sin a}{1+\cos a}$$

$$\sin^2\theta=\frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta=\frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta=\frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\begin{aligned}\cos(a)\cos(b) &= \frac{1}{2}\big(\cos(a+b)+\cos(a-b)\big)\\ \sin(a)\sin(b) &= \frac{1}{2}\big(\cos(a-b)-\cos(a+b)\big)\\ \sin(a)\cos(b) &= \frac{1}{2}\big(\sin(a+b)+\sin(a-b)\big)\\ \cos(a)\sin(b) &= \frac{1}{2}\big(\sin(a+b)-\sin(a-b)\big)\end{aligned}$$

$$\sin(A)+\sin(B)=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin(A)-\sin(B)=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos(A)+\cos(B)=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos(A)-\cos(B)=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\frac{\sin A}{A}=\frac{\sin B}{B}=\frac{\sin C}{C}$$

$$c^2=a^2+b^2-2ab\cos C$$

$$x=r\cos\theta\,,y=r\sin\theta\,,x^2+y^2=r^2,\tan^{-1}\left(\frac{y}{x}\right)=\theta$$

$$a+bi=r(\cos\theta+i\sin\theta)$$

$$(a+b)^n=\sum\binom{n}{i}a^{n-i}b^i$$