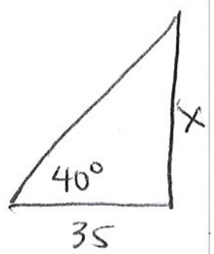
MAT 166, Exam #3, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. At a certain time of day, the angle of elevation of the sun is $40°$. To the nearest foot, find the height of a tree whose shadow is 35 feet long. (4 points)



$\tan 40° = \frac{x}{35}$

$35 \tan 40° = 29.4$ ft

2. Find the exact value of the six trig functions if the coterminal side of the angle passes through the point $(-1, -3)$. (4 points)

x y

$(-1)^2 + (-3)^2 = \sqrt{10}$

$\sin\theta = \frac{-3}{\sqrt{10}}$

$\cos\theta = \frac{-1}{\sqrt{10}}$

$\tan\theta = 3$

$\cot\theta = \frac{1}{3}$

$\sec\theta = -\sqrt{10}$

$\csc\theta = \frac{-\sqrt{10}}{3}$

3. Use a reference angle to find the exact value of each of the following. (3 points each)

a. $\sin\left(-\frac{35\pi}{6}\right)$

$\frac{1}{2}$

b. $\tan 210°$

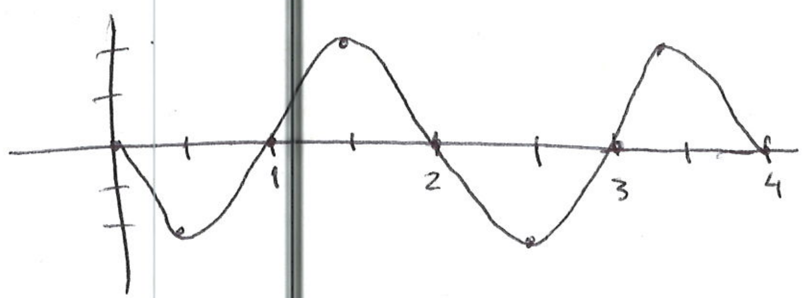
$\frac{1}{\sqrt{3}}$

4. Use key points to graph two periods of each function, by hand, using key points. (6 points each)

a. $y = -2\sin \pi x$

x	0	$\pi/2$	π	$3\pi/2$	2π
$\sin x$	0	1	0	-1	0

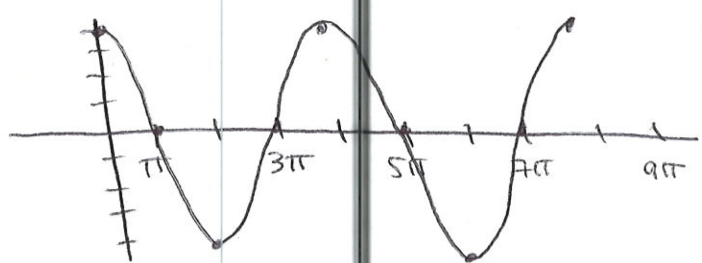
x	0	$1/2$	1	$3/2$	2
$-2\sin \pi x$	0	-2	0	2	0



b. $y = 4\cos\frac{1}{2}x$

x	0	$\pi/2$	π	$3\pi/2$	2π
$\cos x$	1	0	-1	0	1

x	0	π	2π	3π	4π
$4\cos\frac{1}{2}x$	4	0	-4	0	4



5. For each function, state the amplitude, period, phase shift and any vertical shift. (3 points each)

a. $y = -\frac{1}{2}\cos(2x + \pi)$

amplitude $= \frac{1}{2}$ phase shift $-\frac{\pi}{2}$

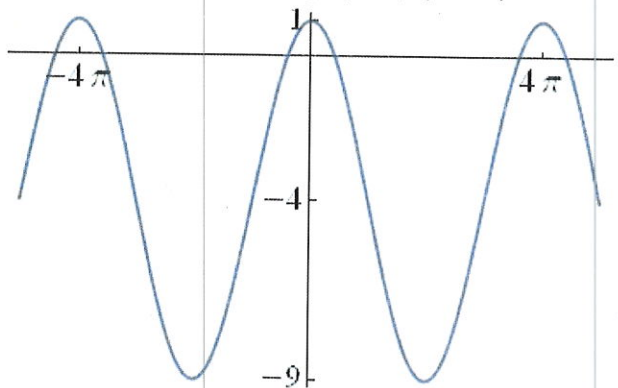
period $T = \frac{2\pi}{2} = \pi$

b. $y = -3\sin 2\pi x + 2$

amplitude $= 3$ phase shift $= 0$

period $T = \frac{2\pi}{2\pi} = 1$

6. Write an equation of the graph. (4 points)



$y = 5\cos\left(\frac{1}{2}x\right) - 4$

$1-(-9) = 10$

amplitude $= \frac{10}{2} = 5$

vertical shift $= -4$

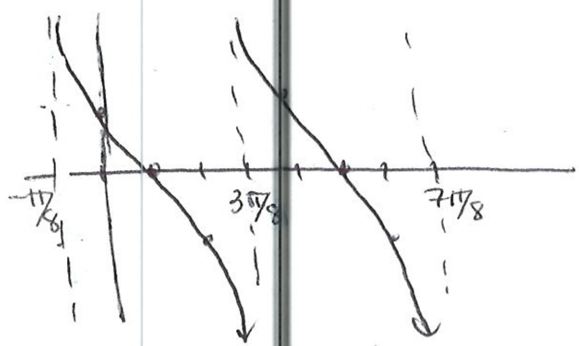
period : $4\pi = \frac{2\pi}{\omega} \rightarrow \omega = \frac{1}{2}$

7. Graph the functions for 2 periods, by hand, using key points. State the domain of each. (5 points)

a. $y = -\tan\left(2x - \frac{\pi}{4}\right)$

x	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$
$\tan x$	und.	-1	0	1	und.

x	$-\pi/4$	$-\pi/8$	0	$\pi/8$	$\pi/4$
$+\pi/4$ x	$-\pi/8$	0	$\pi/8$	$\pi/4$	$3\pi/8$
$-\tan(2x-\pi/4)$	und	1	0	-1	und



domain $x \neq \frac{(4k+3)\pi}{8}$

8. Find the exact value of each expression. (3 points each)

a. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$5\pi/6$

b. $\tan^{-1}(1)$

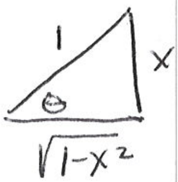
$\pi/4$

c. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$-\pi/4$

d. $\tan(sin^{-1}(x))$

$= \dfrac{x}{\sqrt{1-x^2}}$



9. Find the domain and range of $y = \sin^{-1}(x-2) + \frac{\pi}{2}$. (4 points)

$[-1,1] \rightarrow [1,3]$ domain

$[-\pi/2, \pi/2] \rightarrow [0,\pi]$ range

10. Verify the identities. (4 points each)

a. $(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}$

$(\sec x - \tan x)(\sec x - \tan x) =$

$\sec^2 x - 2\sec x \cdot \tan x + \tan^2 x =$

$1 + \tan^2 x + \tan^2 x - 2\sec x \tan x =$

$1 + 2\tan^2 x - 2\sec x \tan x$

$1 + \dfrac{2\sin^2 x}{\cos^2 x} - \dfrac{2\sin x}{\cos^2 x} = \dfrac{\cos^2 x + 2\sin^2 x - 2\sin x}{\cos^2 x} =$

$\dfrac{1 + \sin^2 x - 2\sin x}{\cos^2 x} = \dfrac{(1-\sin x)^2}{1-\sin^2 x} = \dfrac{(1-\sin x)^{\cancel{2}}}{\cancel{(1-\sin x)}(1+\sin x)} = \dfrac{1-\sin x}{1+\sin x}$

b. $\sin^4 t - \cos^4 t = 1 - 2\cos^2 t$

$$(\sin^2 t - \cos^2 t)\underbrace{(\sin^2 t + \cos^2 t)}_{=1} =$$

$$1 - \cos^2 t - \cos^2 t = 1 - 2\cos^2 t$$

c. $\sin\left(x + \frac{3\pi}{2}\right) = -\cos x$

$$\sin(x)\underbrace{\cos(3\pi/2)}_{0} + \underbrace{\sin(3\pi/2)}_{-1}\cos(x)$$

$$= -1 \cos x$$

d. $\sin^2 \frac{\theta}{2} = \frac{\sec\theta - 1}{2\sec\theta}$

$$\sin\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2} \cdot \frac{\sec\theta}{\sec\theta} = \frac{\sec\theta - 1}{2\sec\theta}$$

e. $\sin 2t - \tan t = \tan t \cos 2t$

$$2\sin t\cos t - \tan t =$$

$$2\sin t\cos t - \frac{\sin t}{\cos t} = \frac{\sin t}{\cos t}(2\cos^2 t - 1) =$$

$$\tan t \cos 2t$$

11. Use identities to find exact values for each of the following. (3 points each)

a. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}+\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$$\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)=\cos\left(\frac{4\pi}{12}\right)=\cos\left(\pi/3\right)=\frac{1}{2}$$

b. $\sin 75°$

$$\sin(45°+30°)=\sin(45°)\cos(30°)+\sin 30°\cos 45°$$

$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{1}{2}\frac{\sqrt{2}}{2}=\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

c. $\cos 22.5°$

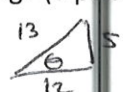
$$\sqrt{\frac{1+\cos 45°}{2}}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}\cdot\frac{\frac{1}{2}}{\frac{1}{2}}}=\sqrt{\frac{1}{2}+\frac{\sqrt{2}}{4}}=\sqrt{\frac{2+\sqrt{2}}{4}}=\boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

d. $\tan\frac{7\pi}{8}$

$$\frac{\sin(7\pi/4)}{1+\cos(7\pi/4)}=\frac{\frac{-1}{\sqrt{2}}}{1+1/\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{-\sqrt{2}}{\sqrt{2}+1}\cdot\frac{\sqrt{2}-1}{\sqrt{2}-1}=\frac{-2+\sqrt{2}}{2-1}=\boxed{-2+\sqrt{2}}$$

12. For $\cos\theta=\frac{12}{13}$, and θ in Q IV, find each of the following. (3 points each)

a. $\sin 2\theta$



$$=2\sin\theta\cos\theta$$

$$2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right)=\frac{-120}{169}$$

b. $\tan\frac{\theta}{2}$

$$\frac{-5/13}{1+\frac{12}{13}}\cdot\frac{13}{13}=\frac{-5}{13+12}=\frac{-5}{25}=-\frac{1}{5}$$

13. Solve for all values of the variable in $[0,2\pi)$. (5 points each)

a. $\sin 2x = \sin x$

$2\sin x\cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$\boxed{x = 0, \pi \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}}$

b. $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sin\alpha = \frac{\sqrt{2}}{2}$

$\alpha = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

$2x - \frac{\pi}{4} = \frac{\pi}{4}$

$2x = \frac{\pi}{2}$

$\boxed{x = \frac{\pi}{4}}$

$2x - \frac{\pi}{4} = \frac{3\pi}{4}$

$2x = \pi$

$\boxed{x = \frac{\pi}{2}}$

$2x - \frac{\pi}{4} = \frac{9\pi}{4}$

$2x = \frac{5\pi}{2}$

$\boxed{x = \frac{5\pi}{4}}$

$2x - \frac{\pi}{4} = \frac{11\pi}{4}$

$2x = 3\pi$

$\boxed{x = \frac{3\pi}{2}}$

c. $3\cos^2 x = \sin^2 x$

$3\cos^2 x - \sin^2 x = 0$

$3\cos^2 x - (1 - \cos^2 x) = 0$

$4\cos^2 x - 1 = 0$

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm\frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

Some useful formulas:

$$\sin(a+b) = \sin a\cos b + \sin b\cos a$$

$$\sin(a-b) = \sin a\cos b - \sin b\cos a$$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

$$\begin{aligned} \sin 2\alpha &= 2\sin\alpha\cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha} \end{aligned}$$

$$\sin(\frac{a}{2}) = \pm\sqrt{\frac{(1-\cos a)}{2}}$$

$$\cos(\frac{a}{2}) = \pm\sqrt{\frac{(1+\cos a)}{2}}$$

$$\tan(\frac{a}{2}) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$