MAT 202, Quiz #9, Fall 2016 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Use the Gram-Schmidt process to create an orthogonal basis for the space spanned by $\left\{\begin{bmatrix}-1\\1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\1\\0\end{bmatrix}, \begin{bmatrix}2\\2\\-1\\1\end{bmatrix}\right\}$.

u_1 u_2 u_3

$\vec{v}_1 = \begin{bmatrix}-1\\1\\0\\1\end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix}2\\1\\1\\0\end{bmatrix} - \left(\frac{-2+1+0+0}{1+1+0+1}\right)\begin{bmatrix}-1\\1\\0\\1\end{bmatrix} =$

$\begin{bmatrix}2\\1\\1\\0\end{bmatrix} + \frac{1}{3}\begin{bmatrix}-1\\1\\0\\1\end{bmatrix} = \begin{bmatrix}5/3\\4/3\\1\\1/3\end{bmatrix} \Rightarrow \vec{v}_2 = \begin{bmatrix}5\\4\\3\\1\end{bmatrix}$

$\vec{v}_3 = \begin{bmatrix}2\\2\\-1\\1\end{bmatrix} - \left(\frac{-2+2+0+1}{1+1+0+1}\right)\begin{bmatrix}-1\\1\\0\\1\end{bmatrix} - \left(\frac{10+8-3+1}{25+16+9+1}\right)\begin{bmatrix}5\\4\\3\\1\end{bmatrix} =$

$\begin{bmatrix}2\\2\\-1\\1\end{bmatrix} - \frac{1}{3}\begin{bmatrix}-1\\1\\0\\1\end{bmatrix} - \frac{16}{51}\begin{bmatrix}5\\4\\3\\1\end{bmatrix} = \begin{bmatrix}13/17\\7/17\\-33/17\\6/17\end{bmatrix}$ $\vec{v}_3 = \begin{bmatrix}13\\7\\-33\\6\end{bmatrix}$

Orthogonal basis $\left\{\begin{bmatrix}-1\\1\\0\\1\end{bmatrix}, \begin{bmatrix}5\\4\\3\\1\end{bmatrix}, \begin{bmatrix}13\\7\\-33\\6\end{bmatrix}\right\}$

2. If a matrix is 7×6, can the linear transformation defined by the matrix be
   a. One-to-one?

yes, there can be a pivot in every column

   b. Onto?

no, there are 7 rows but only 6 columns so there cannot be 7 pivots

   c. Can the range of the matrix be equal to the codomain?

no, since the codomain is $\mathbb{R}^7$ and Rank cannot be bigger than 6.

3. Determine if the transformation $T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{pmatrix}x_1 + 2 + x_2\\x_3 - x_1\\x_2 + 4x_3\end{pmatrix}$. If it is linear, prove it. If it is not, find a counterexample.

it is not linear since $T\left(\begin{bmatrix}0\\0\\0\end{bmatrix}\right) = \begin{bmatrix}2\\0\\0\end{bmatrix}$

(it will violate $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ and $T(c\vec{u}) = cT(\vec{u})$ also)