MAT 202, Quiz #8, Fall 2016 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. For each case below, find $\langle u, v\rangle$, $\|u\|$, $\|u-v\|$, and the angle between the vectors. For the vectors in R^n use the standard dot product. For the vectors in P_n use the inner product given by $\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)dx$.

a. $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ $\quad \vec{u}\cdot\vec{v} = 2-2+0 = 0$

$\|\vec{u}\| = \sqrt{1+4+1} = \sqrt{6}$

$\vec{u}-\vec{v} = \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}$ $\quad \|\vec{u}-\vec{v}\| = \sqrt{1+9+1} = \sqrt{11}$

angle: $\cos\theta = \dfrac{0}{\sqrt{6}\sqrt{5}} = 0 \qquad \theta = \pi/2$

b. $f(x) = 1-2x, g(x) = x^2$

$\langle f,g\rangle = \int_{-1}^{1}(1-2x)x^2\,dx = \int_{-1}^{1} x^2 - \underbrace{\cancel{2x^3}}_{\text{odd}\to 0}\,dx = 2\int_0^1 x^2\,dx$ (even) $= 2\cdot\frac{1}{3}x^3\Big|_0^1 = \boxed{\frac{2}{3}}$

$\|f\| = \sqrt{\int_{-1}^{1}(1-2x)(1-2x)\,dx} = \sqrt{\int_{-1}^{1} 1-\underbrace{\cancel{4x}^{0}}_{\text{odd}}+4x^2\,dx} = \sqrt{2\int_0^1 1+4x^2\,dx}$ (even)

$\sqrt{2(x+\frac{4}{3}x^3\Big|_0^1} = \sqrt{2(1+\frac{4}{3})} = \boxed{\sqrt{14/3}}$

$\|f-g\| = \sqrt{\int_{-1}^{1}(1-2x-x^2)^2\,dx} = \sqrt{\int_{-1}^{1}x^4+\underbrace{4x^3+2x^2-4x}_{\text{odd}}+1\,dx} = \sqrt{2\int_0^1 x^4+2x^2+1\,dx}$ (even)

$= \sqrt{(\frac{1}{5}x^5+\frac{2}{3}x^3+x\Big|_0^1 \cdot 2} = \sqrt{\frac{56}{15}}$

$\|g\| = \sqrt{\int_{-1}^{1}x^4\,dx} = \sqrt{2(\frac{1}{5}x^5\Big|_0^1}$ (even) $= \sqrt{2/5}$

$\cos\theta = \dfrac{(2/3)}{\sqrt{56/15}\cdot\sqrt{2/5}} = \dfrac{5\sqrt{21}}{42}$

$\approx 56.94°$

$\theta \approx .993757$ rads