MAT 201, Quiz #9, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find $\frac{dy}{dx}$ using partial derivatives for $\tan^{-1}(x^2y) = x + xy^2$.

$F(x,y) = \tan^{-1}(x^2y) - x - xy^2$

$F_x = \frac{1}{1+x^4y^2} \cdot 2xy - 1 - y^2 = \frac{2xy}{1+x^4y^2} - 1 - y^2$

$F_y = \frac{1}{1+x^4y^2} \cdot x^2 - 2xy = \frac{x^2}{1+x^4y^2} - 2xy$

$\frac{dy}{dx} = \dfrac{\frac{2xy}{1+x^4y^2} - 1 - y^2}{\frac{x^2}{1+x^4y^2} - 2xy} = \dfrac{2xy - 1 - x^4y^2 - y^2 - x^4y^4}{x^2 - 2xy - 2x^5y^3}$

2. Find the Jacobian for the change of variables over the region $y = \frac{1}{x}, y = \frac{4}{x}, y = x, y = 4x$.

$xy = 1 \quad xy = 4 \qquad \frac{y}{x} = 1 \quad \frac{y}{x} = 4 \qquad u \in [1,4]$

$u = xy \qquad v = \frac{y}{x} \Rightarrow vx = y \qquad v \in [1,4]$

$u = x(vx) = x^2v \Rightarrow \frac{u}{v} = x^2 \Rightarrow x = \sqrt{\frac{u}{v}} = u^{1/2}v^{-1/2}$

$y = v \cdot \sqrt{\frac{u}{v}} = \sqrt{uv} = u^{1/2}v^{1/2}$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2}u^{-1/2}v^{-1/2} & -\frac{1}{2}u^{1/2}v^{-3/2} \\ \frac{1}{2}u^{-1/2}v^{1/2} & \frac{1}{2}u^{1/2}v^{-1/2} \end{vmatrix} = \frac{1}{4}v^{-1} + \frac{1}{4}v^{-1} = \boxed{\frac{1}{2v}}$

3. Find $\vec{\nabla}f$ for $f(x,y) = xy - 2x - 2y - x^2 - y^2$. Graph the equations $f_x = 0, f_y = 0$ and sketch the direction field. Find the critical point(s) and determine if each is a maximum, minimum or saddle point (or cannot be determined). Verify your result with the second partials test.

$\langle y - 2 - 2x, x - 2 - 2y \rangle \qquad f_{xx} = -2 \quad f_{yy} = -2$

$y = 2x + 2 \qquad x = 2 + 2y \qquad f_{xy} = 1$

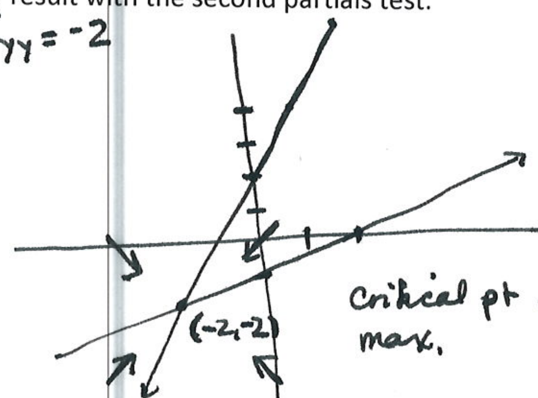
$\frac{x-2}{2} = 2y \Rightarrow y = \frac{1}{2}x - 1$

$y = 2(2+2y) + 2 \qquad x = 2 + 2(-2)$

$y = 4 + 4y + 2 \qquad x = 2 - 4 = -2$

$-4y \quad -4y \qquad (-2,-2)$

$\frac{-3y}{-3} = \frac{6}{-3} \quad y = -2$



Critical pt is a max.

$\vec{\nabla}f(0,0) = \langle -2,-2 \rangle \quad \vec{\nabla}f(0,-3) = \langle -,+ \rangle$

$\vec{\nabla}f(-3,0) = \langle +,- \rangle \quad \nabla f(-4,-4) \langle +,+ \rangle$

$D = (-2)(-2) - 1^2 = 4 - 1 = 3$

$f_{xx} < 0$ max.